3. Three coplanar forces are applied to a box in an attempt to slide it across the floor, as shown. If the box remains at rest, what is the friction force between the box and the floor?

4. A 15 kg flowerpot hangs from wires as shown. Find the tension in wires AB and AC.

4.6 STRESS AND STRAIN

If the vector sum of the external forces acting on a body is zero, the body is in a state of static equilibrium. There are also internal forces acting on the body. Internal forces are caused by external forces, but internal forces do not affect the equilibrium of the body. So, if internal forces do not affect equilibrium, how are they important? To illustrate the importance of internal forces, let's use a familiar example from the sport of weight lifting. See Figure 4.23. As a weight lifter holds a heavy set of weights, his body and the weights are momentarily in a state of static equilibrium. The gravitational force of the weights is balanced by the force exerted on the bar by the weight lifter's hands or shoulders, and the gravitational forces of the weights plus his body are balanced by the force exerted by the floor on his feet. Are there internal forces acting on the weight lifter? Most definitely, yes. If the weight he is holding is great, he is painfully aware of those internal forces. The external forces of the weights and the reaction at the floor cause internal forces in his arms, torso and legs. The magnitude of these internal forces usually limit the time the weight lifter can sustain his position to only a few seconds. Like the weight lifter, engineering structures such as buildings, bridges, and machines experience internal forces when external forces are applied to them. Engineering structures,
however, must usually sustain internal forces for long periods of time, perhaps years. Principles of statics alone, which yield the external forces acting on a body, are insufficient to define the mechanical state of the body. In order for an engineer to make a complete assessment of the structural integrity of any body, internal forces must be considered. From the internal forces and the deformations resulting from them, stress and strain are determined.

![Image of a weight lifter](image.png)

**Figure 4.23.** A weight lifter is in equilibrium, but his body is in a state of stress. (Art by Kathryn Hagen)

### 4.6.1 Stress

The concept of stress is of prime importance in mechanics of materials. Stress is the primary physical quantity that engineers use to ascertain whether a structure can withstand the external forces applied to it. By finding stresses, engineers have a standard method of comparing the ability of a given material to withstand external forces. There are two types of stress, *normal stress* and *shear stress*. In this book, we will confine our attention to normal stress. Normal stress is the *stress that acts normal (perpendicular) to a selected plane or axis within a body*. Normal stress is often associated with the stress in the axial direction in long slender members such as rods, beams, and columns. Consider the slender bar shown in Figure 4.24. An axial force, $P$, acts on each end of the bar, maintaining the bar in equilibrium, as indicated in Figure 4.24a. Now, suppose that we pass an imaginary plane through the bar perpendicular to its axis, as shown in Figure 4.24b. Conceptually, we then remove the bottom portion of the bar that was "cut away" by the imaginary plane. In removing the bottom portion of the bar, we also removed the force applied at the bottom end of the bar that balanced the force applied at the top end. To restore equilibrium, we must apply an equivalent force, $P$, at the "cut
end. This force, unlike the external force applied at the top of the bar, is an internal force because it acts within the bar. This internal force, \( P \), acts perpendicular to the cross-sectional area created by passing the imaginary plane through the bar, as indicated in Figure 4.24c. The normal stress, \( \sigma \), in the bar is defined as the internal force, \( P \), divided by the cross-sectional area, \( A \):

\[
\sigma = \frac{P}{A}
\]

(4.23)

This mathematical definition of normal stress is actually an average normal stress because there may be a variation of stress across the cross section of the bar. Stress variations are normally present only near points where the external forces are applied, however, so Equation 4.23 may be used in the majority of stress calculations without regard to stress variations. The cross section of the bar in Figure 4.24 is circular, but the quantity, \( A \), represents the cross-sectional area of a member of any shape; that is, circular, rectangular, triangular, etc. Note that the definition of stress is very similar to that of pressure. Both quantities are defined as a force divided by an area. Accordingly, stress has the same units as pressure. Typical units for stress are kPa or MPa in the SI system and psi or ksi in the English system.

\[\text{Figure 4.24.} \quad \text{Normal stress in a rod}\]

In Figure 4.24, the force vectors are directed away from each other, indicating that the bar is stretched. The normal stress associated with this force configuration is referred to as tensile stress because the forces place the body in tension. Conversely, if the force vectors are directed toward each other, the bar is compressed. The normal stress associated with this force configuration is referred to as compressive stress.
because the forces place the body in compression. These two force configurations are illustrated in Figure 4.25. One may think that the type of normal stress, tensile or compressive, does not matter because Equation 4.23 says nothing about direction. However, some materials can withstand one type of stress more readily than the other. For example, concrete is stronger in compression than in tension. Consequently, concrete is typically used in applications where the stresses are compressive, such as columns that support bridge decks and highway overpasses. When concrete members are designed for applications involving tensile stresses, reinforcing bar is used.

![Figure 4.25. External forces for tension and compression](image)

4.6.2 Strain

External forces are responsible for producing stress, and they are also responsible for producing deformation. Deformation may also be caused by temperature changes. **Deformation** is defined as a change in the size or shape of a body. No material is perfectly rigid, so when external forces are applied to a body, the body changes its size or shape according to the magnitude and direction of the external forces applied to it. We have all stretched a rubber band and noticed that its length changes appreciably under a small tensile force. All materials—steel, concrete, wood, and other structural materials—deform to some extent under applied forces, but the deformations are usually too small to detect visually, so special measuring instruments are employed. Consider the bar shown in Figure 4.26. Prior to applying an external force, the bar has a length, \( L \). Now, the bar is placed in tension, applying an external force, \( F \), at each end. The tensile force causes the bar to increase in length by an amount, \( \delta \). The quantity, \( \delta \), is called the **normal deformation** or **axial deformation** because the change in length is normal to the direction of the force, which is along the axis of the rod. Depending on the bar's material and the magnitude of the applied force, the normal deformation may be small, perhaps only a few thousandths of an inch. In order to normalize the change in size or shape of a body with respect to the body's original geometry, engineers use a quantity called **strain**. There are two types of strain, **normal strain** and **shear strain**. In this book, we confine our attention to normal strain. Normal strain, \( \varepsilon \), is defined as the **normal deformation**, \( \delta \), divided by the original length, \( L \):

\[
\varepsilon = \frac{\delta}{L} \tag{4-24}
\]

Because strain is a ratio of two lengths, it is a dimensionless quantity. It is customary, however, to express strain as a ratio of two length units. In the SI unit system, strain is usually expressed in units of \( \mu m/m \) because, as mentioned before, deformations are typically small. In the English unit system, strain is usually expressed in units of in/in. Because strain is a dimensionless quantity, strain is sometimes expressed as a percentage.
The normal strain illustrated in Figure 4.26 is for a body in tension, but the definition given by Equation 4.24 also applies to bodies in compression.

\[ e = \frac{\delta}{L} \]

**Figure 4.26.** Normal strain in a rod

### 4.6.3 Hooke’s Law

About three centuries ago, the English mathematician, Robert Hooke (1635 – 1703), discovered that the force required to stretch or compress a spring is proportional to the displacement of a point on the spring. The law describing this phenomena, known as Hooke’s law, is expressed mathematically as

\[ F = k x \]  \hspace{2cm} (4-25)

where \( F \) is force, \( x \) is displacement and \( k \) is a constant of proportionality called the spring constant. Equation 4-25 applies only if the spring is not deformed beyond its ability to resume its original length after the force is removed. A more useful form of Hooke’s law for engineering materials has the same mathematical form as Equation 4-25, but is expressed in terms of stress and strain:

\[ \sigma = E e \]  \hspace{2cm} (4-26)

Hooke’s law given by Equation 4-26 states that the stress, \( \sigma \), in a material is proportional to the strain, \( e \). The constant of proportionality, \( E \), is called the **modulus of elasticity** or **Young’s modulus**, after the English mathematician, Thomas Young (1773 – 1829). Like the spring equation, the engineering version of Hooke’s law applies only if the material is not deformed beyond its ability to resume its original size after the force is removed. A material that obeys Hooke’s law is said to be **elastic** because it returns to its original size after the removal of the force. Because strain, \( e \), is a dimensionless quantity, modulus of elasticity, \( E \), has the same units as stress. Equation 4-26 describes a straight line with \( E \) as the slope. The modulus of elasticity is an experimentally derived quantity. A sample of the material in question is subjected to tensile stresses in a special apparatus that facilitates a sequence of stress and strain measurements in the elastic range of the material. Stress-strain data points are plotted on a linear scale, and a best fit straight line is drawn through the points. The slope of this line is the modulus of elasticity, \( E \).

A useful relationship may be obtained by combining Eqs. (4.23), (4.24), and (4.26). The axial deformation, \( \delta \), may be expressed directly in terms of the internal force, \( F \), and the geometrical and material properties of the member. This is done by substituting the definition of strain, \( e \), given by Equation 4-24 into Equation 4-26.
Hooke's law, and noting that normal stress is the internal force divided by the cross-sectional area, Equation 4-23. The resulting expression is

$$\delta = \frac{PL}{AE}$$  \hspace{1cm} (4-27)

Equation 4-27 is useful because the strain does not have to be calculated first to find the deformation of the member. Equation 4-27 is valid only over the linear region of the stress-strain curve; i.e., only up to the proportional limit of the material.

### 4.6.4 Stress-Strain Diagram

A **stress-strain diagram** is a graph of stress as a function of strain in a given material. The shape of this graph varies somewhat with material, but stress-strain diagrams have some common features. A typical stress-strain diagram is illustrated in Figure 4.27. The upper stress limit of the linear relationship described by Hooke's law is called the **proportional limit**, labeled point A. At any stress between point A and the **elastic limit**, labeled point B, stress is not proportional to strain, but the material will still return to its original size after the force is removed. For many materials, the proportional and elastic limits are very close together. Point C is called the **yield stress** or **yield strength**. Any stress above the yield stress will result in **plastic** deformation of the material; i.e., the material will not return to its original size but will deform permanently. As the stress increases beyond the yield stress, the material experiences a large increase in strain for a small increase in stress. At about point D, called the **ultimate stress** or **ultimate strength**, the cross-sectional area of the material begins to decrease rapidly until the material experiences **fracture** at point E.

![Stress-Strain Diagram](image)

**Figure 4.27.** A typical stress-strain diagram

In the following example, we use the general analysis procedure of:
(1) problem statement, (2) diagram, (3) assumptions, (4) governing equations, (5) calculations, (6) solution check and (7) discussion.
EXAMPLE 4.6:  PROBLEM STATEMENT

A 200-kg engine block hangs from a system of cables as shown in Figure 4.28. Find the normal stress and axial deformation in cables AB and AC. The cables are 0.7 m long and have a diameter of 4 mm. The cables are steel with a modulus of elasticity of $E = 200$ GPa.

![Diagram of suspended engine block for Example 4.6]

**Figure 4.28.** Suspended engine block for Example 4.6

**DIAGRAM**

We will presume that the statics portion of the problem has been solved, so a free-body diagram of the entire system is unnecessary. Diagrams showing a cross section of the cables and the corresponding internal forces are sufficient. (See Figure 4.29.)

![Diagram of cables for Example 4.6]

**Figure 4.29.** Cables for Example 4.6

**ASSUMPTIONS**

1. Cables are circular in cross section.
2. Cables have the same modulus of elasticity.
3. Stress is uniform in the cables.
GOVERNING EQUATIONS

Cross-sectional area:

\[ A = \frac{\pi D^2}{4} \]

Normal stress:

\[ \sigma = \frac{P}{A} \]

Axial deformation:

\[ \delta = \frac{PL}{AE} \]

CALCULATIONS

The cross-sectional area of the cables is

\[ A = \frac{\pi D^2}{4} = \frac{\pi(0.004 \text{ m})^2}{4} = 1.2566 \times 10^{-5} \text{ m}^2 \]

From a prior statics analysis, the tensions in cables AB and AC are 2338 N and 3052 N, respectively. Hence, the normal stress in each cable is

\[ \sigma_{AB} = \frac{P_{AB}}{A} = \frac{2338 \text{ N}}{1.2566 \times 10^{-5} \text{ m}^2} = 186.1 \times 10^6 \text{ N/m}^2 = 186.1 \text{ MPa} \]

\[ \sigma_{AC} = \frac{P_{AC}}{A} = \frac{3052 \text{ N}}{1.2566 \times 10^{-5} \text{ m}^2} = 242.9 \times 10^6 \text{ N/m}^2 = 242.9 \text{ MPa} \]

The deformation in each cable is

\[ \delta_{AB} = \frac{P_{AB}L}{AE} = \frac{(2338 \text{ N})(0.7 \text{ m})}{(1.2566 \times 10^{-5} \text{ m}^2)(200 \times 10^6 \text{ N/m}^2)} = 6.51 \times 10^{-4} \text{ m} = 0.651 \text{ mm} \]

\[ \delta_{AC} = \frac{P_{AC}L}{AE} = \frac{(3052 \text{ N})(0.7 \text{ m})}{(1.2566 \times 10^{-5} \text{ m}^2)(200 \times 10^6 \text{ N/m}^2)} = 8.50 \times 10^{-4} \text{ m} = 0.850 \text{ mm} \]
SOLUTION CHECK

One way to check the validity of the results is to compare the relative magnitudes of the stress and deformation in each cable. The internal force in cable AC is greater than the normal stress in cable AB. Consequently, the normal stress and axial deformation in cable AC must also be greater because the cables are geometrically and materially identical. Our calculations show that this is indeed the case.

DISCUSSION

The deformations are small, less than a millimeter in both cables. These deformations would probably not be significant in an engine hoist application, and would not be perceptible by the naked eye. Are the stresses excessive? Will they plastically deform the cables? To answer these questions, we must know something about the yield stress of the cable material and the stresses for which the cables were designed.

PRACTICE!

In the following practice problems, use the general analysis procedure of:
(1) problem statement, (2) diagram, (3) assumptions, (4) governing equations, (5) calculations, (6) solution check and (7) discussion.

1. A solid rod of stainless steel ($E = 190$ GPa) is 50 cm in length and has a 4 mm x 4 mm cross section. The rod is subjected to an axial tensile force of 5 kN. Find the normal stress, strain, and axial deformation.

2. A 25-cm long, 10 gage wire of yellow brass ($E = 105$ GPa) is subjected to an axial tensile force of 1.75 kN. Find the normal stress and deformation in the wire. A 10 gage wire has a diameter of 2.588 mm.

3. A 8-m high granite column sustains an axial compressive load of 500 kN. If the column shortens 0.12 mm under the load, what is the diameter of the column? For granite, $E = 70$ GPa.

4. A solid rod with a length and diameter of 1 m and 5 mm, respectively, is subjected to an axial tensile force of 20 kN. If the axial deformation is measured as $\delta = 1$ cm, what is the modulus of elasticity of the material?

5. A plastic ($E = 3$ GPa) tube with an outside and inside diameter of 6 cm and 5.4 cm, respectively, is subjected to an axial compressive force of 12 kN. If the tube is 25 cm long, how much does the tube shorten under the load?

4.7 DESIGN STRESS

Most engineering structures are not designed to deform permanently or fracture. Every member in a structure must maintain a certain degree of dimensional control to assure that it does not plastically deform thereby losing its size or shape, interfering with surrounding structures or other members in the same structure. Obviously, the members must not fracture either because this would lead to a catastrophic failure that would result in material and financial loss and perhaps the loss of human life. Therefore, members in most structures are designed to sustain a maximum stress that is below the yield stress on the stress-strain diagram for the particular material used to construct that member. This maximum stress is called the design stress or allowable stress. When a properly designed member is subjected to a load, the stress in the member will not exceed the design stress. Because the design stress is within the elastic range of the material, the member will return to its original dimensions after the load is removed. A bridge, for example, sustains stresses in its members while traffic passes over it. When there is no traffic, the members in the bridge return to their original size. Similarly, while a boiler is operating, the pressure vessel sustains stresses that deform it, but when the pressure is reduced to atmospheric pressure, the vessel returns to its original dimensions.
If a structural member is designed to carry stresses below the yield stress, how does an engineer choose what the allowable stress should be? And why choose a stress below the yield stress in the first place? Why not design the member using the yield stress itself since that would allow the member to carry the maximum possible load? Engineering design is not an exact science. If it was, structures could be designed with ultimate precision using the yield stress, or any other stress for that matter, as the design stress, and the design stress would never be exceeded while the structure was in service. Because design is not an exact science, engineers incorporate an allowance in their designs that takes into account the following uncertainties:

1. **Loadings** The design engineer may not anticipate every type of loading or the number of loadings that may occur. Vibration, impact or accidental loadings may occur that were not accounted for in the design of the structure.

2. **Failure modes** Materials can fail by one or more of several different mechanisms. The design engineer may not have anticipated every failure mode by which the structure can possibly fail.

3. **Material properties** Physical properties of materials are subject to variations during manufacture, and there are experimental uncertainties in their numerical values. Properties may also be altered by heating or deformation during manufacture, handling, and storage.

4. **Deterioration** Exposure to the elements, poor maintenance or unexpected natural phenomena may cause a material to deteriorate, thereby compromising its structural integrity. Various types of corrosion are the most common forms of material deterioration.

5. **Analysis** Engineering analysis is a critical part of design, and analysis involves making simplifying assumptions. Thus, analytical results are not precise but are approximations.

To account for the uncertainties listed above, engineers use a design or allowable stress based on a parameter called the **factor of safety**. The factor of safety (F.S.) is defined as the **ratio of the failure stress to the allowable stress**:

\[
F.S. = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}} \tag{4-28}
\]

Because the yield stress is the stress above which a material plastically deforms, the yield stress, \(\sigma_y\), is commonly used as the failure stress, \(\sigma_{\text{fail}}\). The ultimate stress, \(\sigma_u\), may also be used. The failure stress is always greater than the allowable stress, so \(F.S. > 1\). The value chosen for the factor of safety depends on the type of engineering structure, the relative importance of the member compared to other members in the structure, the risk to property and life and the severity of the design uncertainties listed above. For example, the factor of safety for aircraft and spacecraft structures is typically close to 1 to minimize weight. However, the factor of safety for ground-based structures such as dams, bridges and buildings may be higher, perhaps 1.5 or 2. High-risk structures that pose a safety hazard to people in the event of failure, such as certain nuclear power plant components, may have a factor of safety as high as 3. Factors of safety for structural members in specific engineering systems have been standardized through many years of testing and industrial evaluation. Factors of safety are often defined by building codes or engineering standards established by city, state or federal agencies and professional engineering societies.
APPLICATION: DESIGNING A TURNBuckle

Turnbuckles are special mechanical fasteners that facilitate connections by cables, chains, or cords. A basic turnbuckle consists of a slender, cylindrical shaped body threaded on each end to accept an eye-bolt, a hook, or other type of tying component. The tension in the cables that are tied to a turnbuckle is adjusted by rotating the body of the turnbuckle. Turnbuckles are designed such that tightening or loosening may be accomplished without twisting the cables. Like the cables that are connected to them, turnbuckles must sustain the tensile stresses to which they are subjected. Consider a turnbuckle used to adjust the tension in a cable that stabilizes a communications tower. From a prior analysis, the tension in the cable is determined to be 25 kN. The loaded turnbuckle is shown in Figure 4.30a. Let us suppose that as a new engineer your first job is to select a turnbuckle for this application. Turnbuckles are available in a variety of sizes and materials from several suppliers. Hardware suppliers specify the maximum recommended load that a particular turnbuckle can sustain without failing. It is therefore a simple matter for you, the end user, to select a turnbuckle with a recommended maximum load that is somewhat greater than the actual load of 25 kN. But how did the engineers who designed the turnbuckle obtain this load value? The following example shows how fundamental concepts of stress and factor of safety may be used to design the eye-bolt portion of a turnbuckle. The general analysis procedure of: (1) problem statement, (2) diagram, (3) assumptions, (4) governing equations, (5) calculations, (6) solution check, and (7) discussion, is used.

Figure 4.30. A loaded turnbuckle

PROBLEM STATEMENT
Determine the minimum diameter eye-bolt in a turnbuckle used to stabilize a communications tower. The tensile force in the cable is 25 kN. The eye-bolt is to be made of AISI 4130 steel, a high-strength forging steel. (AISI is an abbreviation for American Iron and Steel Institute.) To account for potential high wind loads and other uncertainties, use a factor of safety of 2.0.

DIAGRAM
The internal and external forces acting on the eye-bolt are shown in Figure 4.30b.

ASSUMPTIONS
1. Stress is uniform in the eye-bolt.
2. Stress in the eye-bolt is purely axial.
3. Consider stress in the main body of the eye-bolt only, not the threads.

GOVERNING EQUATIONS
The governing equations for this problem are the cross-sectional area for a circular bolt, the definition of normal stress and factor of safety.
CALCULATIONS

In the third governing equation, we have used the yield stress, \( \sigma_y \), as the failure stress. The yield stress of AISI 4130 steel is 760 MPa. The objective of the analysis is to find the diameter, \( D \), of the eye-bolt required to sustain the applied load. There are three unknown quantities: \( \sigma_\text{f} \), \( A \), and \( D \). Because the three governing equations are not independent, we may combine them algebraically to obtain the diameter, \( D \). Upon substituting Eq. (a) into Eq. (b) and then Eq. (b) into Eq. (c), we obtain:

\[
D = \frac{4 \cdot \sigma_\text{f} \cdot A}{\pi \cdot \sigma_y^2}
\]

\[
= \frac{4(25 \times 10^6 \text{ N})(2.0)}{\pi(760 \times 10^6 \text{ Pa})}
\]

\[
= 9.15 \times 10^{-3} \text{ m} = 0.915 \text{ mm}
\]

SOLUTION CHECK

No errors were found.

DISCUSSION

The minimum eye-bolt diameter that will sustain the applied load with a factor of safety of 2.0 is 9.15 mm. In English units, this diameter is

\[
D = 9.15 \text{ mm} \times \frac{1 \text{ in}}{25.4 \text{ mm}} = 0.360 \text{ in}
\]

Bolts come in standard diameters, and 0.360 in is not a standard size. Bolts are typically available in standard sizes of 1/4-in, 3/16-in, 3/8-in, etc. A 5/16-in (0.3125 in) bolt is too small, so the 3/8-in (0.375 in) should be chosen even though it is slightly larger than required. It should be emphasized that this analysis reflects only a part of the analysis that would be required in the total design of a turnbuckle. Stresses in the threads of the eye-bolt and the turnbuckle body as well as the main body of the turnbuckle itself would also have to be calculated.

PRACTICE!

In the following practice problems, use the general analysis procedure of: (1) problem statement, (2) diagram, (3) assumptions, (4) governing equations, (5) calculations, (6) solution check, and (7) discussion.

1. A rod of aluminum 6061-T6 has a square cross section measuring 0.25 in \( \times \) 0.25 in. Using the yield stress as the failure stress, find the maximum tensile load that the rod can sustain for a factor of safety of 1.5. The yield stress of aluminum 6061-T6 is 240 MPa.

2. A concrete column with a diameter of 60 cm supports a portion of a highway overpass. Using the ultimate stress as the failure stress, what is the maximum compressive load that the column can carry for a factor of safety of 1.25? For the ultimate stress of concrete, use \( \sigma_u = 40 \text{ MPa} \).

3. A column of rectangular cross section constructed from fir timber is subjected to a compressive load of 6 MN. If the width of the column is 12 cm, find the depth required to sustain the load with a factor of safety of 1.6. The ultimate stress of fir is \( \sigma_u = 50 \text{ MPa} \).