Vibration Testing

For vibration testing, you need

- an excitation source
- a device to measure the response
- a digital signal processor to analyze the system response

i) Excitation sources

Typically either instrumented hammers or shakers are used.

Figure 1 shows a **hammer excitation**. Instrumented hammers have a force transducer near their impact tip, so the force imparted to the structure can be measured. Hammer tests are typically easier to set up. The main disadvantages are 1) it is difficult to get consistent hammer hits, and 2) the energy imparted to the structure may not be enough.

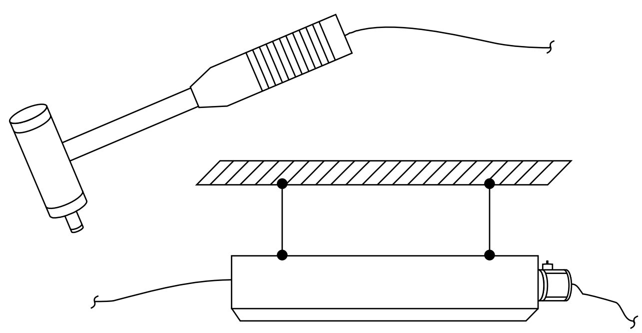

Figure 1. Schematic of impact testing. Note the accelerometer attached on the right end of the structure. Ref. 1

The size of the hammer and hardness of the tip control the duration of the impact and thus the frequency content of the excitation. Time and frequency plots for a soft, medium and hard tip are shown in Figure 2. Note that the softer the tip the longer the impact time. The harder the tip the shorter the impact time. Higher frequencies are needed to represent shorter pulses. That is, the hard tip imparts energy over a broader frequency range than the soft tip as shown in Figure 2b.

[1]The Fundamentals of Modal Testing, Application Note 243-3 Agilent Technologies
[2]Fundamentals of Signal Analysis, Application Note 243 Agilent Technologies

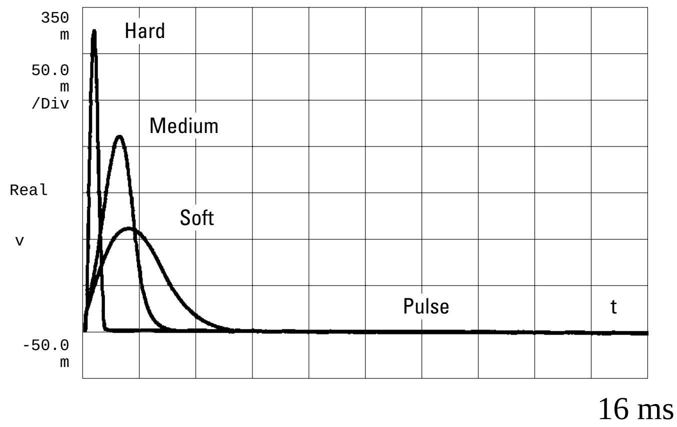

Figure 2a. Measured force versus time comparing soft, medium, and hard hammer tips. Ref. 1

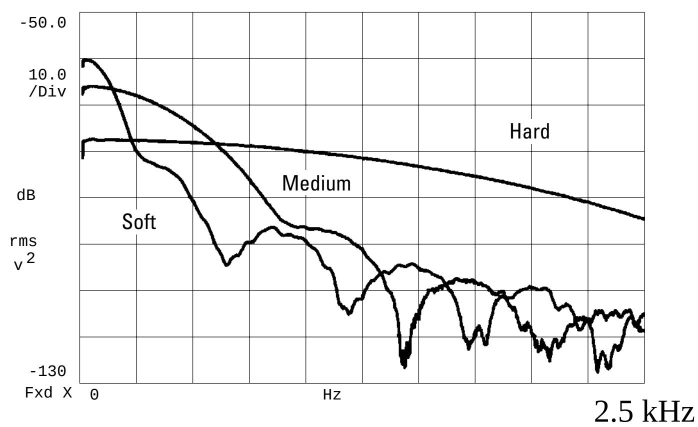

Figure 2a. Measured force versus frequency comparing soft, medium, and hard hammer tips. These plots are obtained by taking the Fourier Transform of the curves in Figure 2a. Ref. 1

This is a good example for thinking about the frequency content of a forcing function.

Figure 3 shows a **shaker excitation**. A controller is needed to control the motion of the shaker. A variety of signals can be created, including 1) single frequencies, 2) a chirp or sine sweep, and 3) random vibration. A rod or wire, called a “stinger” is often used to connect the shaker output to the structure. Often, a force transducer is put between the shaker and the structure, so the excitation force can be measured.

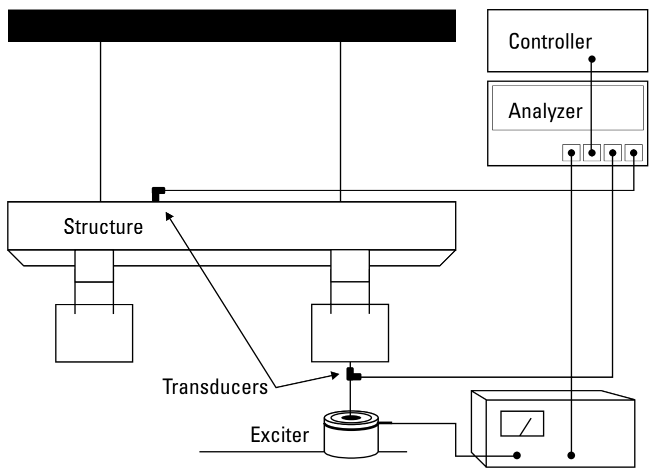

Figure 3. Schematic of test configuration with a shaker as the exciter. Ref. 1

ii) Accelerometers, as shown in Figure 1 and 3, are typically used to measure the response of a system. A basic accelerometer measures the acceleration in one direction. Triaxial accelerometers exist to measure accelerations in 3 perpendicular directions simultaneously. Accelerometers are chosen based on the frequency range of interest and their size. If the mass of the accelerometer is on the order of the mass of the structure, it will affect the vibration. A non-contact device such as a laser vibrometer, capacitance probe, or eddy-current probe, can be used instead to measure the displacement.

iii) A digital signal analyzer is needed to control the shaker and to transform the accelerometer and force transducer data into the frequency space.

Measurements are typically plotted in the frequency domain (Discrete Fourier Transform of the time domain signal) because

- time domain signals are typically complicated and uninformative
- knowledge of the response to harmonic excitation is needed to characterize the structure --- i.e. obtain information about the natural frequencies and mode shapes
- knowledge of the response to harmonic excitation is needed to understand the response to more general loading (which can be very complicated).
- signals important for machine fault diagnosis are easier to identify in the frequency domain.

Because the frequency domain in so important in vibration measurement, we will next discuss the frequency domain representation to a number of signals measured as a function of time. First, look at the time domain signal in figure 4b that consists of 2 sine waves of different frequencies summed together. The 2 sine waves are shown in the lower half of Figure 4b, while the sum is shown as the solid curve above these two sine curves. In Figure 4a, the 2 sine curves are shown in a 3-D plot, with frequency as the 3rd axis. Finally, the plot in Figure 4c is obtained by rotating this 3-D plot. In other words, the time signal consists of 2 frequencies, which are represented as 2 spikes in the frequency domain.

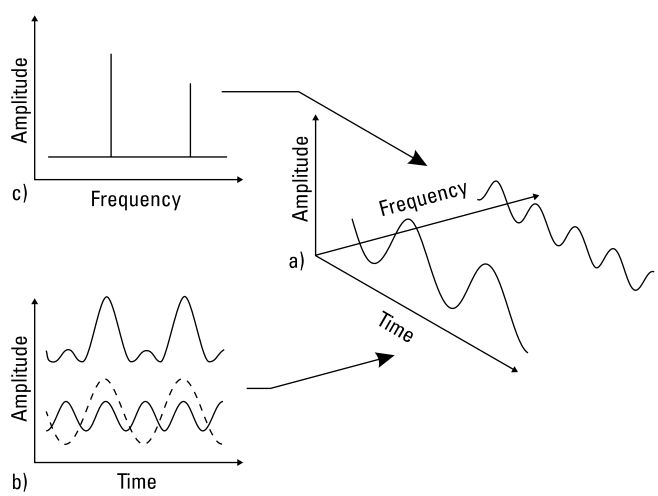

Figure 4. Time and frequency domain representations of a signal obtained by summing 2 sine waves of different frequencies. Ref. 2

As another example, consider the response of a 1-degree-of-freedom system shown schematically in Figure 5 to an arbitrary load f(t). We know the response to a time harmonic excitation can be represented as a plot of amplitude and phase versus forcing frequency as in Figure 7. Note that the frequency domain plot has a peak near the natural frequency. The width of this peak depends on the damping. Recall that we can measure the damping of this 1-degree-of-freedom system from the Quality Factor – which is related to the width of this peak. The more damping the wider the peak.

If the system is driven at a constant frequency, then each point on the curve is equal to the amplitude of the steady state response at that frequency divided by the amplitude of the force. In vibration testing, the plots in Figure 7 can be obtained by dividing the Fourier transform of the response by the Fourier transform of the forcing function for any forcing function. The plots in Figure 7 are thus often called the transfer function.

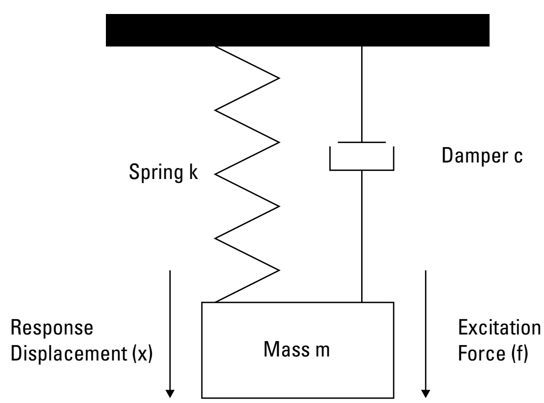

Figure 5. 1-degree-of-freedom system. Ref. 1

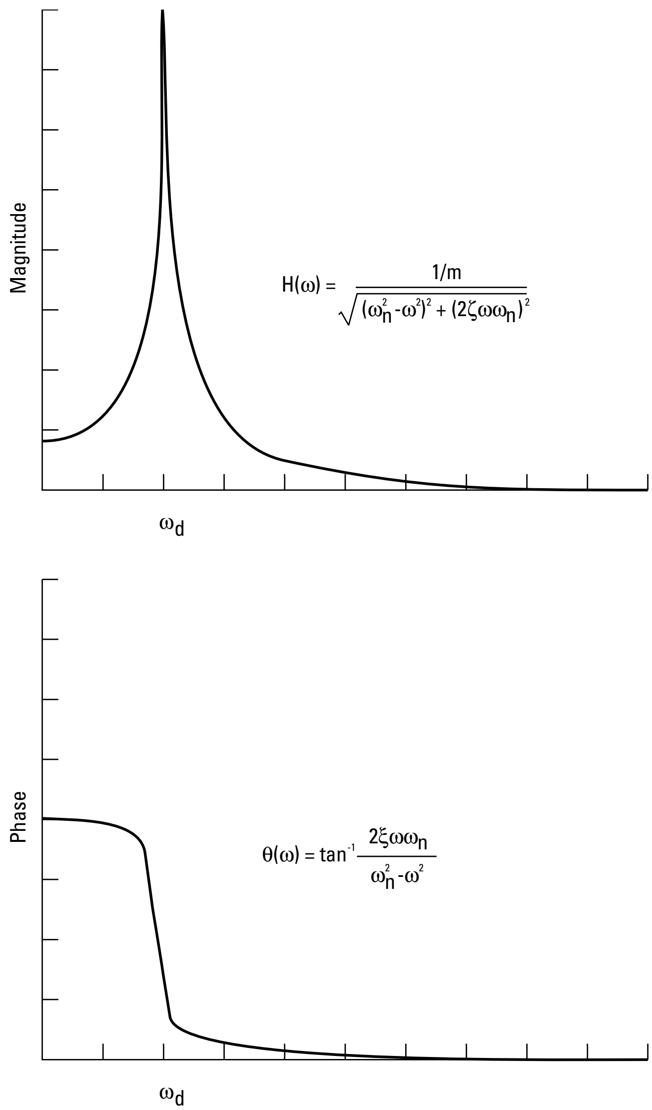

Figure 7. Frequency domain response (amplitude and phase) of a1-degree-of-freedom system to a harmonic loading. Also, transfer function for a1-degree-of-freedom system obtained from an impact loading by dividing the Fourier transform of the response by the Fourier transform of the force. Ref. 1

The response of the one-degree-of-freedom system to a sharp hammer impact is shown in Figure 7. The response is a damped oscillation at the damped natural frequency ω_d. If we take the Fourier transform of the response to a hammer hit and divide by the Fourier transform of the hammer force versus time function, then we should obtain the transfer function in Figure 6.

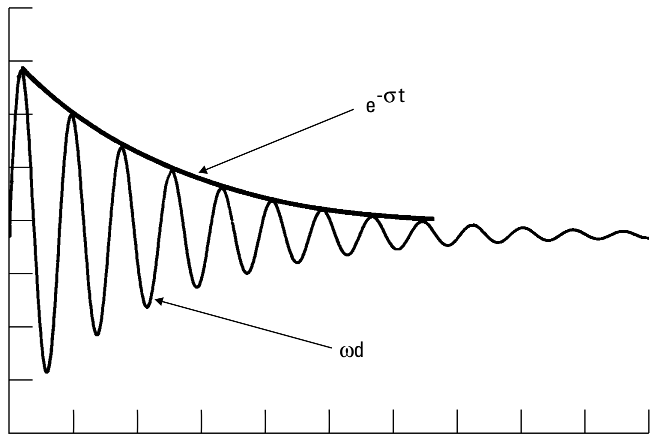

Figure 6. Time domain response of a1-degree-of-freedom system to an impact. Ref. 1

As another example, consider the response of a 3-degree-of-freedom system shown schematically in Figure 8 to a hammer impact. The response, shown in Figure 9, consists of the sum of 3 damped sinusoids. As you can see, it is difficult to make sense of this time domain plot. However, the response is much easier to understand in the frequency domain.

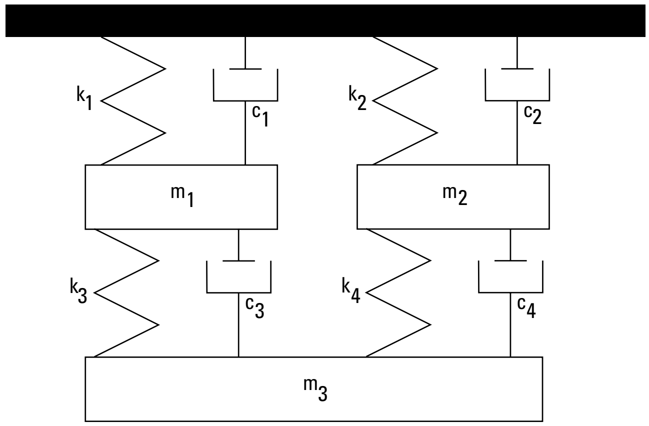

Figure 8. 3-degree-of-freedom system. Ref. 1

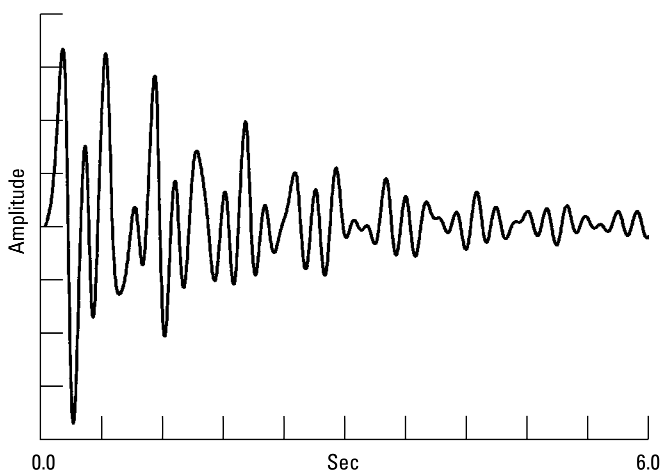

Figure 9. Time domain response of a3-degree-of-freedom system to an impact. Ref. 1

The transfer function in Figure 10a for the 3-degree-of-freedom system has 3 peaks at the 3 natural frequencies. If these peaks are well separated, they can be treated similar to the sole peak of the 1-degree-of-freedom system. Damping of each mode can be obtained from the width of each peak. However, if the peaks overlap, then more sophisticated methods are needed to obtain this information. Figure 10b illustrates the 3 curves, one at each frequency, that are summed to obtain the transfer function of Figure 10a.

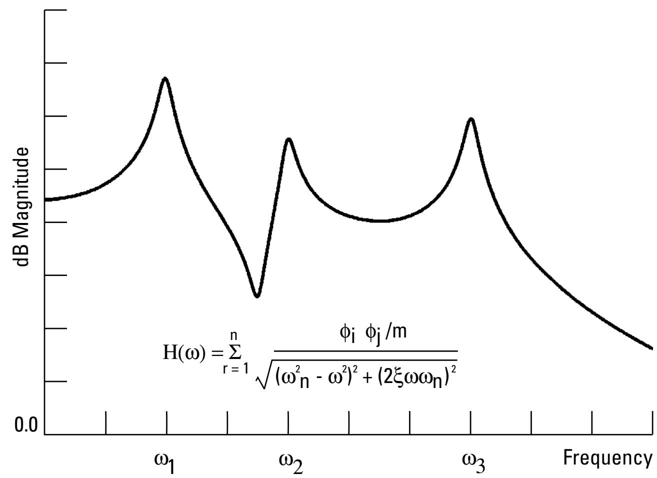

Figure 10a. Frequency domain response of a 3-degree-of-freedom system to a harmonic loading. Also, transfer function for a3-degree-of-freedom system obtained from an impact loading by dividing the Fourier transform of the response by the Fourier transform of the force. Ref. 1

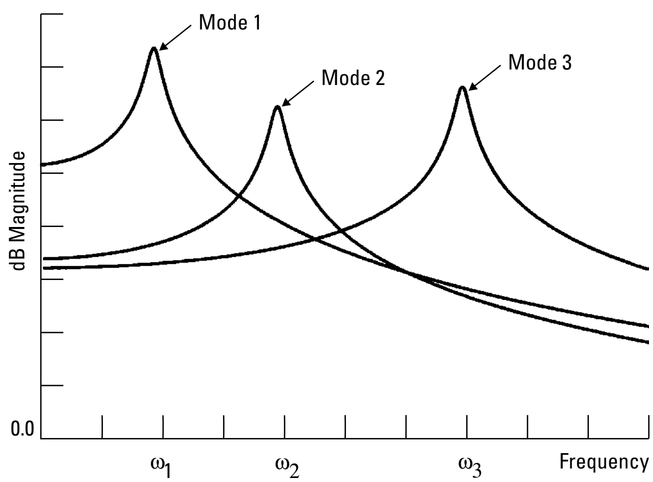

Figure 10b. Individual mode contributions to the 3-degree-of-freedom transfer function shown in Figure 10a. Ref. 1

Measurement Accuracy is affected by a variety of factors. To obtain better accuracy

- Minimize extraneous noise – (electronic, acoustic, floor vibrations, etc.)
- Average over a number of data sets to reduce random noise
- Use a source with sufficient energy input, but keep in the linear range
- Choose an appropriate sampling rate (at least 2X the highest frequency)
- Band-pass filter to prevent aliasing
- Window to avoid errors due to the discrete Fourier transform
- Choose an appropriate input range to match the signal amplitude

THE DISCRETE FOURIER TRANSFORM

All real measurements are discrete sampling of an analog signal, so you must be aware of how this descritization can affect the results. Typically you will choose a constant sampling rate defined by ΔT, the time between samples, and the number of samples N. The total sampling time will therefore be $T_{\max} = N\,\Delta T$. The maximum frequency that can be distinguished $f_{\max}$ and the frequency resolution Δf will be determined by your time domain sampling. This is shown pictorially in Figure 11.

If we define t_o=0, then the Nth and last data point occurs at $t_{N-1} = (N-1)\Delta t$. Because the discrete Fourier transform assumes periodic signals with period $T_{\max}$, $p(T_{\max}) \equiv p(0)$. So the time domain function is discretized into N points as

$$p(t) \Rightarrow \{p(0), p(\Delta t), p(2\Delta t), \ldots, p((N-1)\Delta t)\}.$$

Just as the definition of the Fourier transform differs slightly from book to book, so does the definition of the discrete Fourier transform. In the book by Brigham [Ref. 3], the discrete Fourier transform is defined as

$$P\left(\frac{n}{N\Delta t}\right) = \sum_{k=0}^{N-1} p(k\Delta t) e^{-i2\pi nk/N}, \qquad n = 0,1,2,\ldots,N-1, \tag{1}$$

and the inverse discrete Fourier transform is defined by

$$p(k\Delta t) = \frac{1}{N}\sum_{k=0}^{N-1} P\left(\frac{n}{N\Delta t}\right) e^{i2\pi nk/N}, \qquad k = 0,1,2,\ldots,N-1. \tag{2}$$

From the definitions, it is apparent that the frequency resolution is $\Delta f = 1/(N\Delta t) = 1/T_{\max}$. The discrete Fourier transform produces only N/2 independent ***complex*** numbers, and the second half of the data is just the complex conjugate of the first half, the magnitude symmetric about the mid-frequency point. Therefore, the maximum frequency occurs at n=N/2, that is, $f_{\max} = 1/(2\Delta t)$. But, if N is even, analogous to the time domain, $P(f_{\max}) \equiv P(0)$.

[3]*The Fast Fourier Transform*, E.O. Brigham, Prentice Hall, 1974.

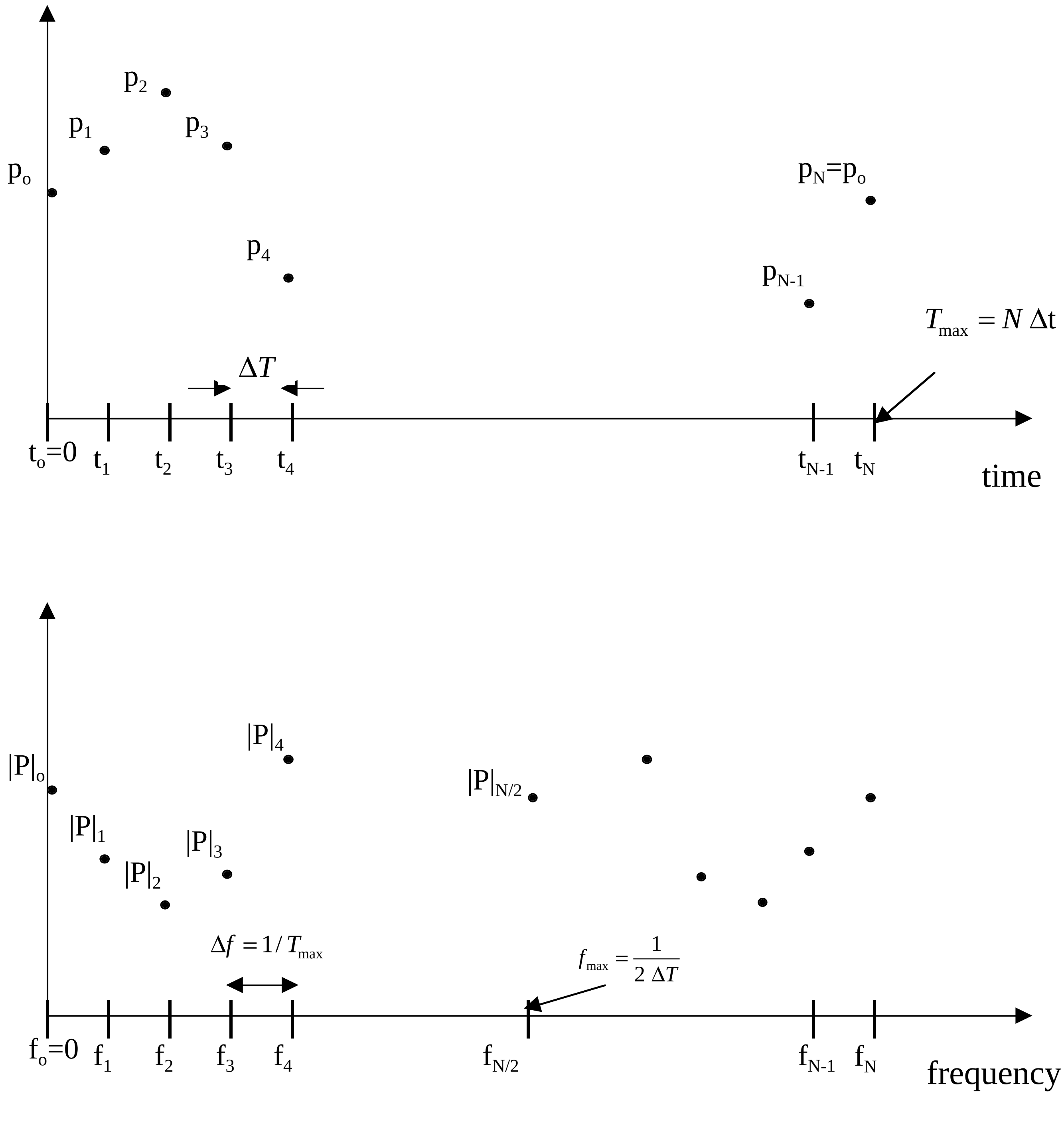

Figure 11. Time domain sampling affects maximum frequency and frequency domain resolution.

Here we repeat the two important relationships between the time and frequency domain sampling.

MAXIMUM FREQUENCY SAMPLED: $f_{\max} = \dfrac{1}{2\,\Delta T}$ (3)

FREQUENCY RESOLUTION: $\Delta f = \dfrac{f_{\max}}{N/2} = \dfrac{1}{T_{\max}}$ (4)

The first equation is a statement of the Nyquist Criterion – the sampling rate $1/\Delta T$ must be at least twice the highest frequency. That is, there must be at least 2 samples per cycle to identify the frequency correctly.

The second equation results from the restriction that there are $N/2$ complex data points in the frequency domain corresponding to the N data points in the time domain. These are demonstrated in Figure11.

If there is significant signal at frequencies higher than $f_{\max}$, the sampling rate will not be adequate to identify the frequency and therefore the high frequency signal will appear to be at a lower frequency. This is demonstrated for a simple time harmonic signal in Figure 12. This is called ***aliasin**g*. Aliasing can be prevented by filtering out the energy at frequencies higher than $f_{\max}$.

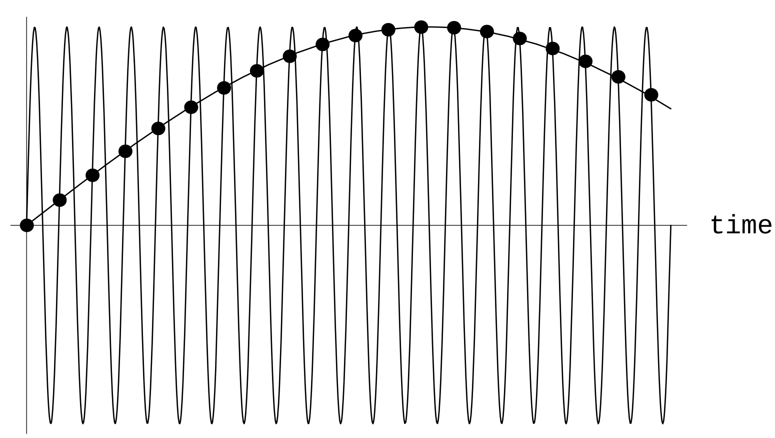

Figure 12. Aliasing – A signal will appear to be of much lower frequency if the sampling rate is not adequate.

If we compare the definition of the discrete Fourier transform to that of the complex Fourier series, we can then determine the amplitude of the contribution at each frequency.

The complex Fourier series of a periodic function $f(t)$ with period $T_1 = 2\pi/\Omega_1$ can be defined as

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{in\Omega_1 t}, \qquad \text{where ,} \qquad F_n = \frac{1}{T_1}\int_0^{T_1} f(t) e^{-in\Omega_1 t}. \qquad (5)$$

The energy at frequency $n\Omega_1$ is equal to $|F_n| + |F_{-n}| = 2|F_n|$. So if we approximate the integral for F_n as a summation of discrete data points, then we obtain

$$F_n = \frac{1}{T_1}\int_0^{T_1} f(t) e^{-in2\pi t/T_1} \approx \frac{1}{N\Delta t}\sum_{k=0}^{N-1} f(k\Delta t) e^{-in2\pi\frac{k\Delta t}{N\Delta t}}\Delta t = \frac{1}{N}\sum_{k=0}^{N-1} f(k\Delta t) e^{-i2\pi nk/N} \qquad (6)$$

By comparing the definition of the Fourier series in Equation (6) to the definition used in Ref. 3 for the discrete Fourier transform in Equation (2), we conclude that the discrete Fourier transform amplitude P_n must be multiplied by $2/N$ to obtain the contribution at the frequency $n\Delta f$.

Mathematica uses a slightly different definition for the discrete Fourier transform pair. Let $\{b_s\}$ represent the list that is the DFT of the list $\{a_r\}$, where $r, s = 1, 2, 3, \ldots, N$. Then the Mathematica commands relating these two lists are

$$\{b_s\} = Fourier[\{a_r\}] \qquad \text{and} \qquad \{a_r\} = InverseFourier[\{b_s\}]. \quad (7)$$

Mathematica's definitions are

$$b_s = \frac{1}{\sqrt{N}}\sum_{r=1}^{N} a_r e^{2\pi i(r-1)(s-1)/N} \qquad \text{and} \qquad a_r = \frac{1}{\sqrt{N}}\sum_{r=1}^{N} b_s e^{-2\pi i(r-1)(s-1)/N}. \quad (8)$$

The only real difference between the definitions in Equations (1) and (2) and the definitions in Equation (8) is that the amplitude of the Mathematica's DFT is $1/\sqrt{N}$ times the definition in Brigham's book. Therefore, we conclude that the discrete Fourier transform amplitude b_s must be multiplied by $2\sqrt{N}/N$ to obtain the contribution at the frequency $(s-1)\Delta f$.

Discrete Fourier transforms assume that the signal is periodic with time period T_{max}. If the signal level and slope at the end does not match the signal level and slope at the beginning, then this effectively will introduce a jump in the signal because of the assumed periodicity of the signal. This is illustrated in Figure 12. ***Windowing*** is used to weight the signal at this artificial discontinuity so that it has less of an effect on the Fourier transform. However, care must be taken when using windows because they can affect the Fourier transforms in other ways. A "Uniform" window uses a constant unity weight factor over the entire domain, i.e., it is equivalent to no window. The Uniform window should be used unless there is a specific reason to window the data.

The effect of using a non-uniform windowing function on a DFT is illustrated in Figure 13 and 14. First, Figure 13 a) shows the time domain signal for a harmonic wave that has the same value and slope at the start and end. For this signal no discontinuity is introduced and the DFT in Figure 13 b) has nonzero values only at the fundamental and the first 2 harmonics. On the other hand, a jump in slope is introduced when the time domain signal in Figure 13 c) is periodically extended, so the DFT of this signal, shown in Figure 13 d) is nonzero over a large range of frequencies. This smearing of the energy throughout the frequency domains is a phenomenon known as leakage. To lessen the effect of the introduced discontinuity, a "window" can be used to weight the contributions from the end values less. A Hanning window is the most typical window used and its use is shown in Figure 12 c) and d). The DFT of the time domain signal in Figure 13 c) after windowing is applied is given in Figure 14. Note that the peak is sharper, but not as sharp as in Figure 13 b). Care must be used in interpreting DFT, since the spreading of the peak in Figure 14 is not due to damping but is due to an artifact of the DFT.

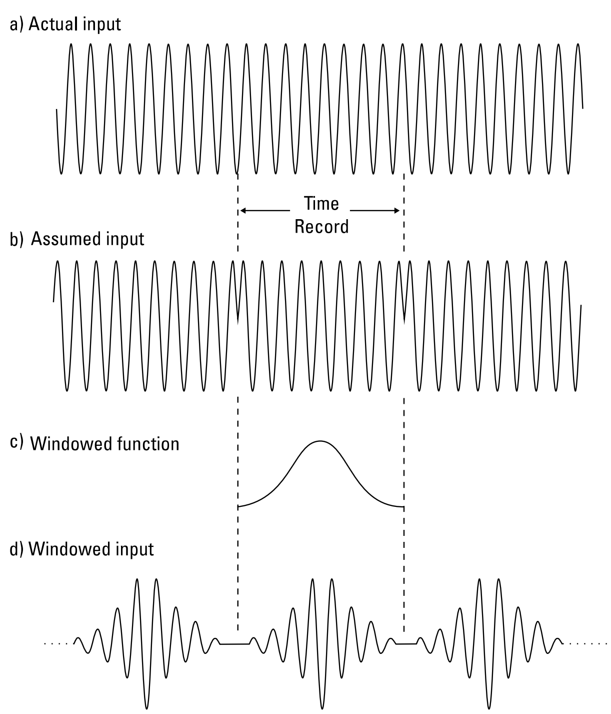

Figure 12. Illustration of windowing a time domain signal. Ref. 1

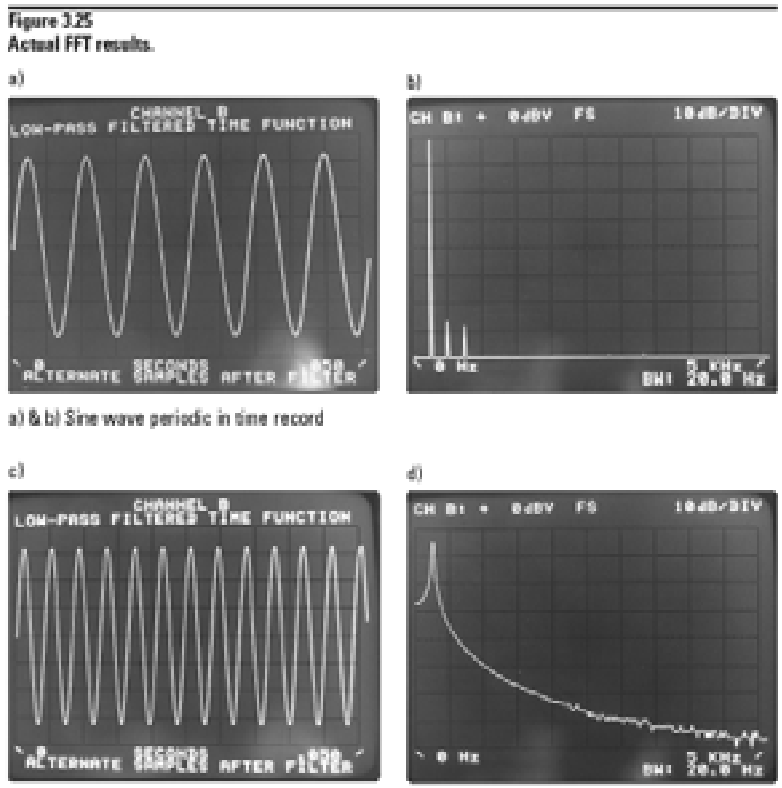

Figure 13. Illustration of the effect of a discontinuity in slope between the start and end of a time-domain signal on the resulting discrete Fourier transform. Ref 2.

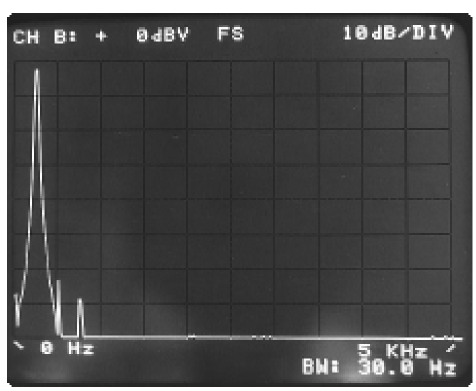

Figure 14. Discrete Fourier transform of the time domain signal in Figure 13 c) after a Hanning window is applied. Ref. 2

A FlatTop windowing function works similar to a Hanning windowing function, but may improve the accuracy of amplitude measurements at the expensive of widening the filter range.

When exciting a structure with an instrumented hammer and measuring the resulting transient signal, the Hanning or FlatTop windows should not be used. As illustrated in Figure 15, the Hanning window would eliminate much of the transient, which has energy over a wide range of frequencies. So, in this case, using the Hanning window would remove information from the DFT.

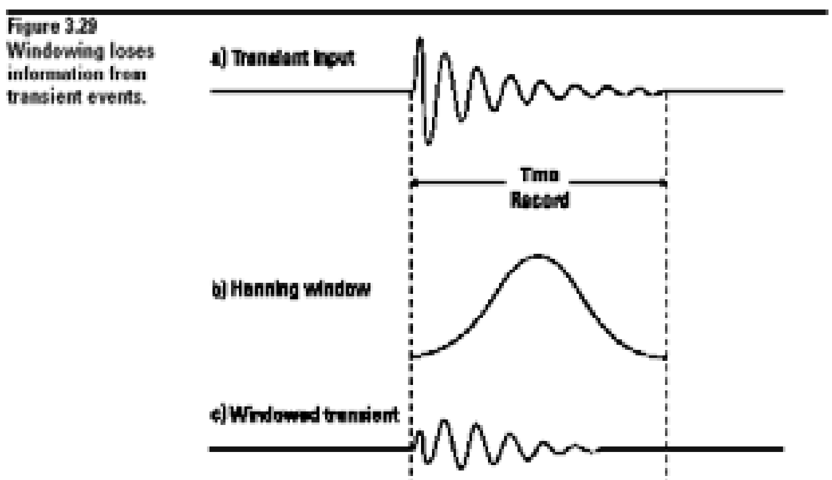

Figure 15. Distortion of a transient signal by a Hanning windowing function. Ref. 2.

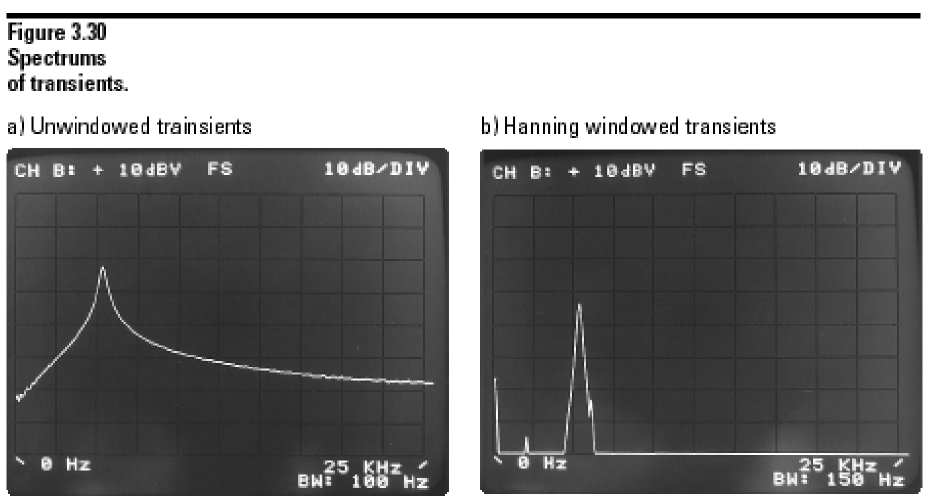

Figure 16. DFT of transient signal of Figure 15, using a a) uniform and b) Hanning windowing function. Ref. 2.

There are windowing functions designed to use with instrumented hammer tests. If the transients in a transient signal decay before $t=T_{max}$ and there is significant background noise that is evident in the measured signal after the transients have decayed, then a Force window can be used on the hammer signal and an Exponential window can be used on the response (e.g. accelerometer) signal. It is important to choose the time span and decay rate of these two windows to have the desired effect. In particular, if the decay rate is too rapid, the exponential window can add extraneous damping to the system. Examples of these two windowing functions are shown in Figures 17 and 18.

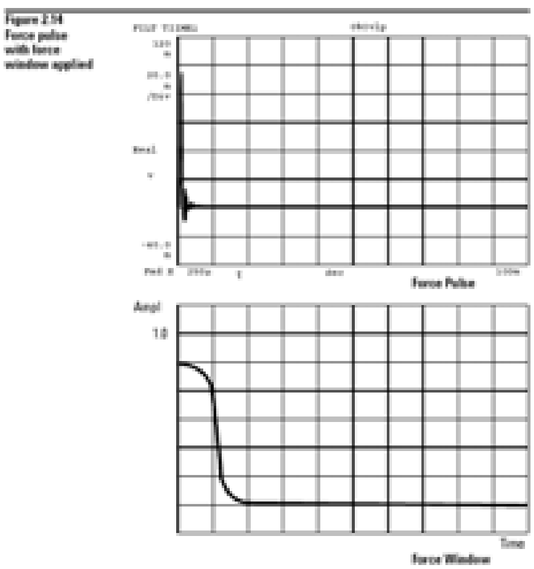

Figure 17. Force window. Ref. 1

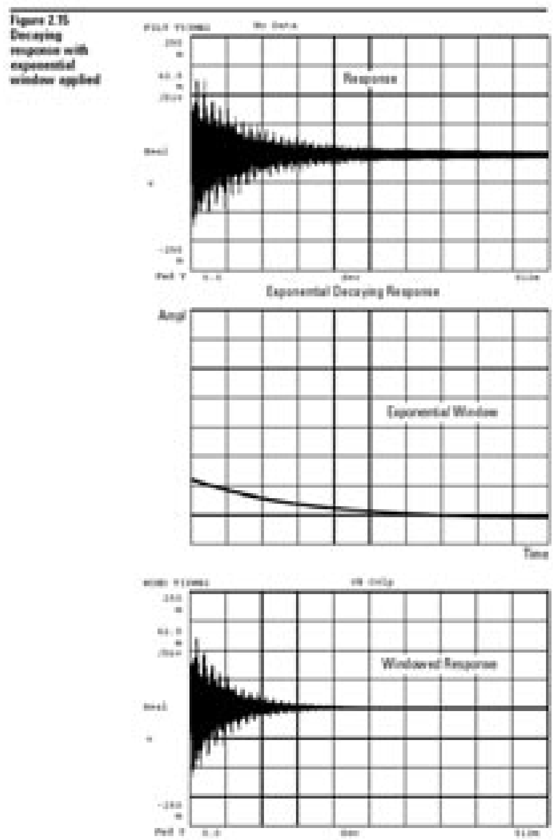

Figure 18. Exponential window. Ref. 1.

The INPUT RANGE is also important to monitor on a digital signal analyzer. Note that the input range is not necessarily the display range, as is typical on an oscilloscope. If the signal uses only a small portion of the input range, then the discritization of the signal amplitude will lead to significant error. Any part of the signal that exceeds the input range will not be correctly represented (this is indicated by the "overload" light). This is demonstrated in the three sets of plots in Figure 19.

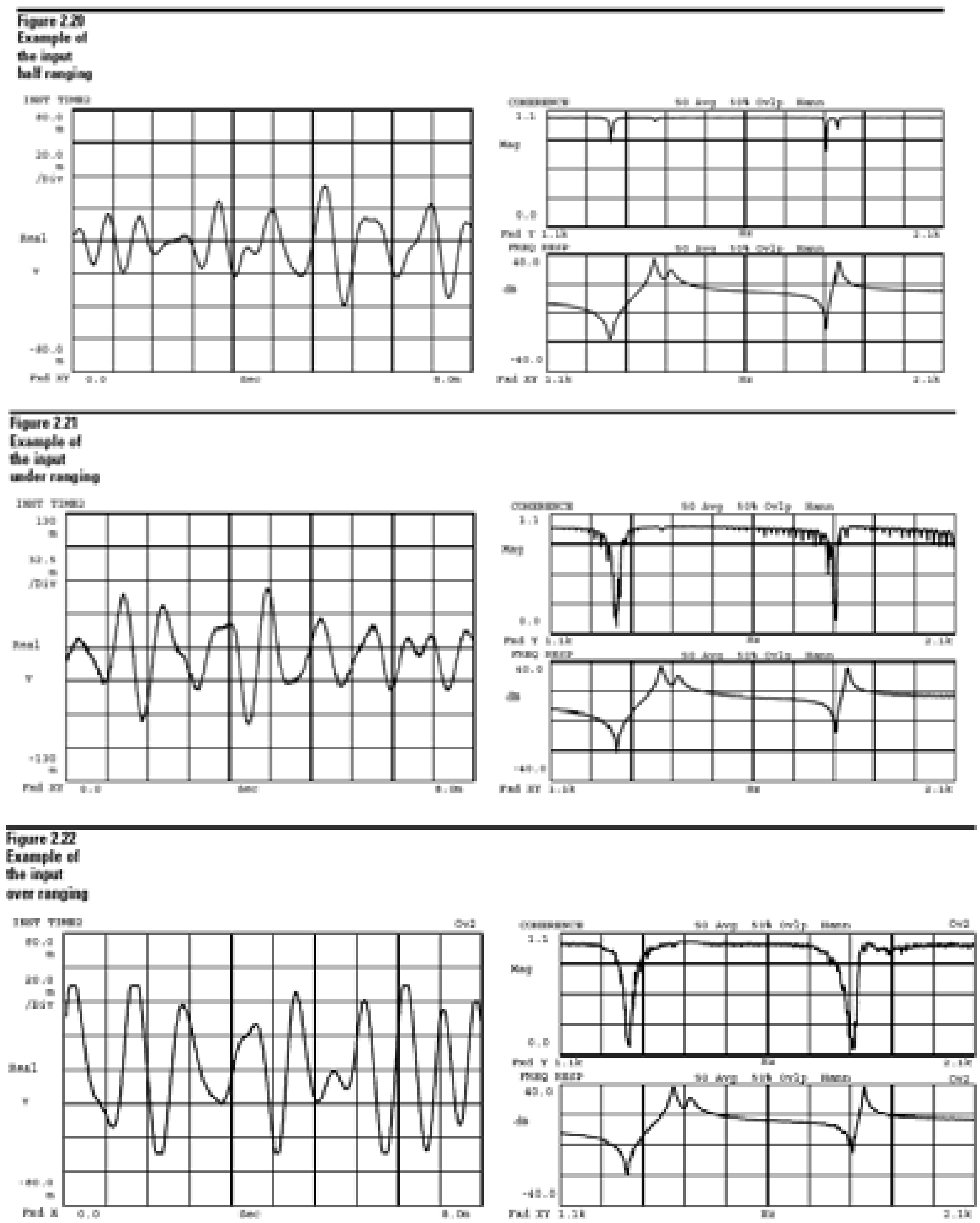

Figure 19. Illustration of the importance of using the correct input range.