C H A P T E R 5

FREQUENCY RESPONSE ANALYSIS

Frequency response analysis is a method used to compute structural response to steady-state oscillatory excitation. Examples of oscillatory excitation include rotating machinery, unbalanced tires, and helicopter blades. In frequency response analysis the excitation is explicitly defined in the frequency domain. All of the applied forces are known at each forcing frequency. Forces can be in the form of applied forces and/or enforced motions (displacements, velocities, or accelerations).

Frequency response analysis is performed in the frequency domain.

Phase Shift

Oscillatory loading is sinusoidal in nature. In its simplest case, this loading is defined as having an amplitude at a specific frequency. The steady-state oscillatory response occurs at the same frequency as the loading. The response may be shifted in time due to damping in the system. The shift in response is called a phase shift because the peak loading and peak response no longer occur at the same time. An example of phase shift is shown in Figure 5-1.

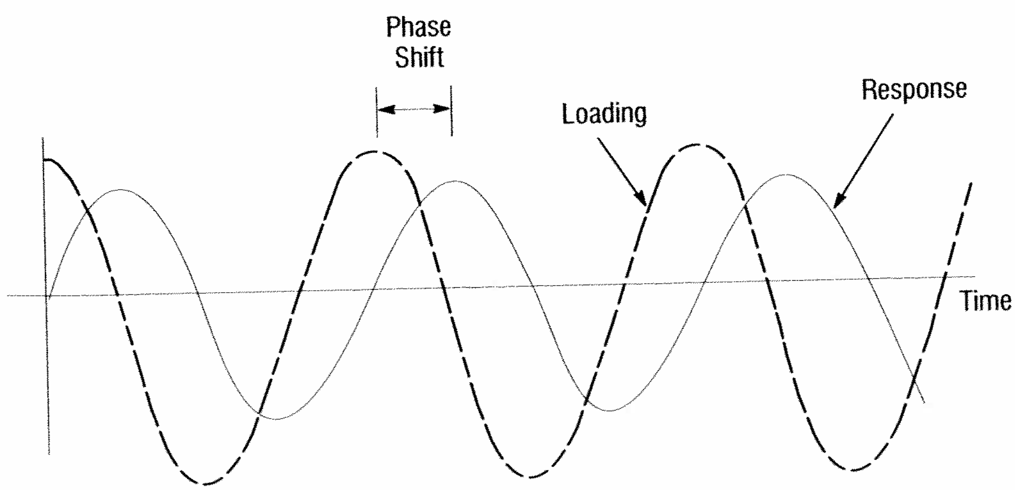

Figure 5-1. Phase Shift.

Complex Numbers

The important results obtained from a frequency response analysis usually include the displacements, velocities, and accelerations of grid points as well as the forces and stresses of elements. The computed responses are complex numbers defined as magnitude and phase (with respect to the applied force) or as real and imaginary components, which are vector components of the response in the real/imaginary plane. These quantities are graphically presented in Figure 5-2.

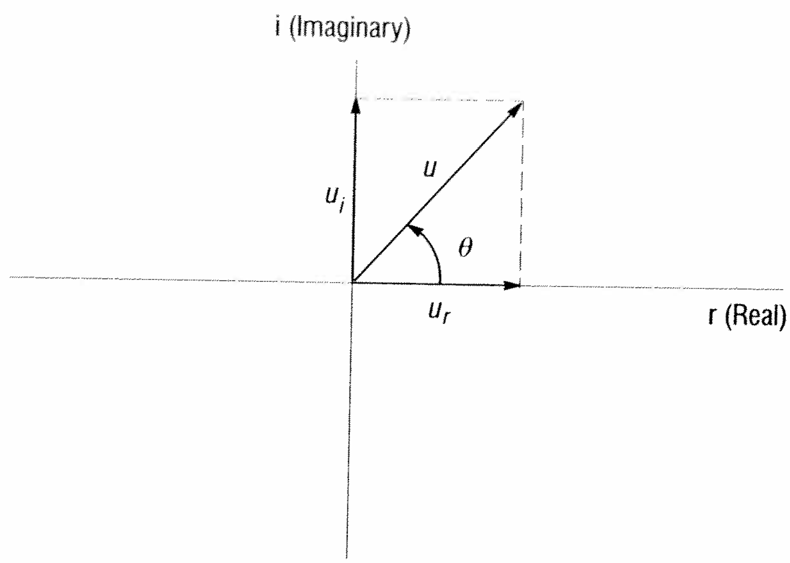

Figure 5-2. Complex Plane.

where u $=$ magnitude $= \sqrt{u_r^2 + u_i^2}$

θ $=$ phase angle $= \tan^{-1}(u_i/u_r)$

u_r $=$ real component $= u \cos\theta$

u_i $=$ imaginary component $= u \sin\theta$

Two different numerical methods can be used in frequency response analysis. The direct method solves the coupled equations of motion in terms of forcing frequency. The modal method utilizes the mode shapes of the structure to reduce and uncouple the equations of motion (when modal or no damping is used); the solution for a particular forcing frequency is obtained through the summation of the individual modal responses. The choice of the method depends on the problem. The two methods are described in Sections 5.1 and 5.2.

5.1 Direct Frequency Response Analysis

In direct frequency response analysis, structural response is computed at discrete excitation frequencies by solving a set of coupled matrix equations using complex algebra. Begin with the damped forced vibration equation of motion with harmonic excitation

$$[M]\{\ddot{x}(t)\} + [B]\{\dot{x}(t)\} + [K]\{x(t)\} = \{P(\omega)\}e^{i\omega t} \qquad (5\text{-}1)$$

The load in Eq. (5-1) is introduced as a complex vector, which is convenient for the mathematical solution of the problem. From a physical point of view, the load can be real or imaginary, or both. The same interpretation is used for response quantities.

For harmonic motion (which is the basis of a frequency response analysis), assume a harmonic solution of the form:

$$\{x\} = \{u(\omega)\}e^{i\omega t} \qquad (5\text{-}2)$$

where $\{u(\omega)\}$ is a complex displacement vector. Taking the first and second derivatives of Eq. (5-2), the following is obtained:

$$\begin{aligned} \{\dot{x}\} &= i\omega\{u(\omega)\}e^{i\omega t} \\ \{\ddot{x}\} &= -\omega^2\{u(\omega)\}e^{i\omega t} \end{aligned} \qquad (5\text{-}3)$$

When the above expressions are substituted into Eq. (5-1), the following is obtained:

$$-\omega^2[M]\{u(\omega)\}e^{i\omega t} + i\omega[B]\{u(\omega)\}e^{i\omega t} + [K]\{u(\omega)\}e^{i\omega t} = \{P(\omega)\}e^{i\omega t} \qquad (5\text{-}4)$$

which after dividing by $e^{i\omega t}$ simplifies to

$$\left[-\omega^2 M + i\omega B + K\right]\{u(\omega)\} = \{P(\omega)\} \qquad (5\text{-}5)$$

The equation of motion is solved by inserting the forcing frequency ω into the equation of motion. This expression represents a system of equations with complex coefficients if damping is included or the applied loads have phase angles. The equations of motion at each input frequency are then solved in a manner similar to a statics problem using complex arithmetic.

5.4 Frequency-Dependent Excitation Definition

An important aspect of a frequency response analysis is the definition of the loading function. In a frequency response analysis, the force must be defined as a function of frequency. Forces are defined in the same manner regardless of whether the direct or modal method is used.

The following Bulk Data entries are used for the frequency-dependent load definition:

RLOAD1	Tabular input—real and imaginary.
RLOAD2	Tabular input—magnitude and phase.
DAREA	Spatial distribution of dynamic load.
LSEQ	Generates the spatial distribution of dynamic loads from static load entries.
DLOAD	Combines dynamic load sets.
TABLEDi	Tabular values versus frequency.
DELAY	Time delay.
DPHASE	Phase lead.

The particular entry chosen for defining the dynamic loading is largely a function of user convenience for concentrated loads. Pressure and distributed loads, however, require a more complicated format.

There are two important aspects of dynamic load definition. First, the location of the loading on the structure must be defined. Since this characteristic locates the loading in space, it is called the spatial distribution of the dynamic loading. Secondly, the frequency variation in the loading is the characteristic that differentiates a dynamic load from a static load. This frequency variation is called the temporal distribution of the load. A complete dynamic loading is a product of spatial and temporal distributions.

Using Table IDs and Set IDs in MSC/NASTRAN makes it possible to apply many complicated and temporally similar loadings with a minimum of input. Combining simple loadings to create complicated loading distributions that vary in position as well as frequency is also a straightforward task.

The remainder of this section describes the Bulk Data entries for frequency-dependent excitation. The description is given in terms of the coefficients that define the dynamic load. See Appendix F for more complete Bulk Data descriptions.

The RLOAD1 entry defines the real and imaginary parts.

Frequency-Dependent Loads – RLOAD1 Entry

The RLOAD1 entry is a general form in which to define a frequency-dependent load. It defines a dynamic loading of the form

$$\{P(f)\} = \left\{A\left[C(f) + iD(f)\right]e^{i\{\theta - 2\pi f \tau\}}\right\} \tag{5-21}$$

The values of the coefficients are defined in tabular format on a TABLEDi entry. You need not explicitly define a force at every excitation frequency. Only those values that describe the character of the loading are required. MSC/NASTRAN will interpolate for intermediate values.

1	2	3	4	5	6	7	8	9	10
RLOAD1	SID	A	τ	θ	C	D			

Field	**Contents**
SID	Set ID defined by a DLOAD Case Control command or a DLOAD Bulk Data entry.
A	Spatial distribution of the load and scale factor (DAREA entry ID).
τ	DELAY entry ID. (Used only with a DAREA entry.)
θ	DPHASE entry ID. (Used only with a DAREA entry.)
C	TABLEDi entry that defines C(f).
D	TABLEDi entry that defines D(f).

Note that f is the frequency in cycles per unit time.

Frequency-Dependent Loads – RLOAD2 Entry

The RLOAD2 entry is a variation of the RLOAD1 entry used for defining a frequency-dependent load. Whereas the RLOAD1 entry defines the real and imaginary parts of the complex load, the RLOAD2 entry defines the magnitude and phase.

The RLOAD2 entry defines dynamic excitation in the form

$$\{P(f)\} = \left\{AB(f)e^{i\{\phi(f)+\theta-2\pi f\tau\}}\right\} \tag{5-22}$$

The RLOAD2 definition may be related to the RLOAD1 definition by

$$\underbrace{C(f) + iD(f)}_{\substack{\text{RLOAD1}\\ \text{Definition}}} = \underbrace{B(f)e^{i\phi(f)}}_{\substack{\text{RLOAD2}\\ \text{Definition}}} \tag{5-23}$$

1	2	3	4	5	6	7	8	9	10
RLOAD2	SID	A	τ	θ	B	ϕ			

Field	Contents
SID	Set ID defined by a DLOAD Case Control command.
A	Spatial distribution of the load and scale factor (DAREA ID).
τ	DELAY entry ID. (Used only with a DAREA entry.)
θ	DPHASE entry ID. (Used only with a DAREA entry.)
B	TABLEDi entry defining amplitude versus frequency pairs.
ϕ	TABLEDi entry defining phase angle versus frequency pairs.

Not that f is the frequency in cycles per unit time.

Spatial Distribution of Loading – DAREA Entry

The DAREA entry defines the degrees of freedom where the dynamic load is to be applied and the scale factor to be applied to the loading. The DAREA entry provides the basic spatial distribution of the dynamic loading.

1	2	3	4	5	6	7	8	9	10
DAREA	SID	P1	C1	A1	P2	C2	A2		

Field	**Contents**
SID	Set ID specified by RLOADi entires.
Pi	Grid, extra, or scalar point ID.
Ci	Component number.
Ai	Scale factor.

A DAREA entry is selected by RLOAD1 or RLOAD2 entries. Any number of DAREA entries may be used; all those with the same SID are combined.

Time Delay – DELAY Entry

The DELAY entry defines the time delay τ in an applied load.

1	2	3	4	5	6	7	8	9	10
DELAY	SID	P1	C1	τ_1	P2	C2	τ_2		

Field	**Contents**
SID	Set ID specified by an RLOADi entry.
Pi	Grid, extra, or scalar point ID.
Ci	Component number.
τ_i	Time delay for Pi, Ci (default = 0.0).

A DAREA entry must be defined for the same point and component. Any number of DELAY entries may be used; all those with the same SID are combined.

Phase Lead – DPHASE Entry

The DPHASE entry defines the phase lead θ.

1	2	3	4	5	6	7	8	9	10
DPHASE	SID	P1	C1	θ_1	P2	C2	θ_2		

Field	Contents
SID	Set ID specified by an RLOADi entry.
Pi	Grid, extra, or scalar point ID.
Ci	Component number.
θ_i	Phase lead (in degrees) for Pi, Ci (default = 0.0).

A DAREA entry must be defined for the same point and component. Any number of DPHASE entires may be used; all those with the same SID are combined.

Dynamic Load Tabular Function – TABLEDi Entries

The TABLEDi entries (i = 1 through 4) each define a tabular function for use in generating frequency-dependent dynamic loads. The form of each TABLEDi entry varies slightly, depending on the value of i, as does the algorithm for y(x). The x values need not be evenly spaced.

The TABLED1, TABLED2, and TABLED3 entries linearly interpolate between the end points and linearly extrapolate outside of the endpoints as shown in Figure 5-4. The TABLED4 entry uses the endpoint values for values beyond the endpoints.

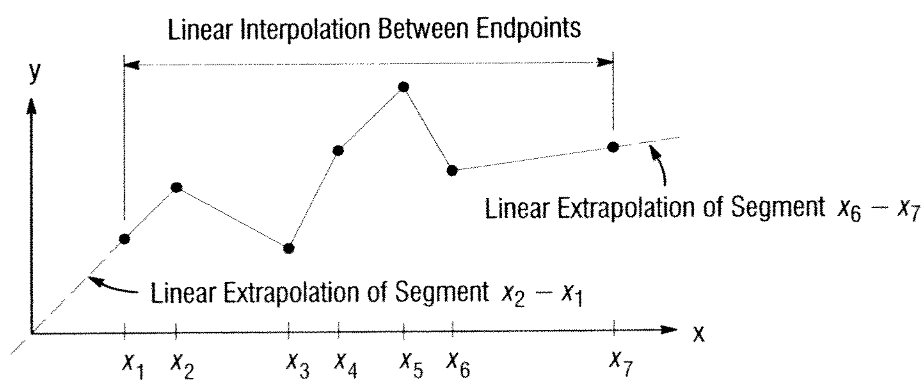

Figure 5-4. Interpolation and Extrapolation for TABLED1, TABLED2, and TABLED3 Entries.

The TABLED1 entry has the following format:

1	2	3	4	5	6	7	8	9	10
TABLED1	TID								
	x1	y1	x2	y2	x3	y3	-etc.-	ENDT	

Field	**Contents**
TID	Table identification number.
xi, yi	Tabular values. Values of x are frequency in cycles per unit time.

The TABLED1 entry uses the algorithm

$$y = y_T(x) \tag{5-24}$$

ENDT ends the table input.

The TABLED2 entry has the following format:

1	2	3	4	5	6	7	8	9	10
TABLED2	TID	X1							
	x1	y1	x2	y2	x3	y3	-etc.-	ENDT	

Field	**Contents**
TID	Table identification number.
X1	Table parameter.
xi, yi	Tabular values. Values of x are frequency in cycles per unit time.

The TABLED2 entry uses the algorithm

$$y = y_T(x - X1) \tag{5-25}$$

ENDT ends the table input.

The TABLED3 entry has the following format:

1	2	3	4	5	6	7	8	9	10
TABLED3	TID	X1	X2						
	x1	y1	x2	y2	x3	y3	-etc.-	ENDT	

Field	Contents
TID	Table identification number.
X1, X2	Table parameters ($X2 \neq 0.0$).
xi, yi	Tabular values. Values of x are frequency in cycles per unit time.

The TABLED3 entry uses the algorithm

$$y = y_T\left(\frac{x - X1}{X2}\right) \tag{5-26}$$

ENDT ends the table input.

The TABLED4 entry has the following format:

1	2	3	4	5	6	7	8	9	10
TABLED4	TID	X1	X2	X3	X4				
	A0	A1	A2	A3	A4	A5	-etc.-	ENDT	

Field	Contents
TID	Table identification number.
Xi	Table parameters ($X2 \neq 0.0$; $X3 < X4$).
Ai	Coefficients.

The TABLED4 entry uses the algorithm

$$y = \sum_{i=0}^{N} Ai\left(\frac{x - X1}{X2}\right)^i \tag{5-27}$$

N is the degree of the power series. When $x < X3$, $X3$ is used for x; when $x > X4$, $X4$ is used for x. This condition has the effect of placing bounds on the table; there is no extrapolation outside of the table boundaries.

There is no extrapolation outside of the table boundaries for the TABLED4 entry.

ENDT ends the table input.

DAREA Example

Suppose the following command is in the Case Control Section:

DLOAD = 35

in addition to the following entries in the Bulk Data Section:

1	2	3	4	5	6	7	8	9	10
$RLOAD1	SID	DAREA	DELAY	DPHASE	TC	TD			
RLOAD1	35	29	31		40				
$DAREA	SID	POINT	COMPONENT	SCALE					
DAREA	29	30	1	5.2					
$DELAY	SID	POINT	COMPONENT	LAG					
DELAY	31	30	1	0.2					
$TABLED1	ID								
$	x1	y1	x2	y2	x3	y3	x4	y4	
TABLED1	40								
	0.0	4.0	2.0	8.0	6.0	8.0	ENDT		

The DLOAD Set ID 35 in Case Control selects the RLOAD1 entry in the Bulk Data having a Set ID 35. On the RLOAD1 entry is a reference to DAREA Set ID 29, DELAY Set ID 31, and TABLED1 Set ID 40. The DAREA entry with Set ID 29 positions the loading on grid point 30 in the 1 direction with a scale factor of 5.2 applied to the load. The DELAY entry with Set ID 31 delays the loading on grid point 30 in the 1 direction by 0.2 units of time. The TABLED1 entry with Set ID 40 defines the load in tabular form. This table is shown graphically in Figure 5-5. The result of these entries is a dynamic load applied to grid point 30, component T1, scaled by 5.2 and delayed by 0.2 units of time.

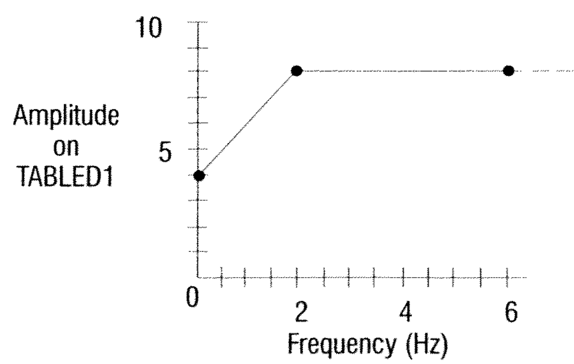

Figure 5-5. TABLED1 – Amplitude Versus Frequency.

5.5 Solution Frequencies

A major consideration in a frequency response analysis is selecting the frequencies at which the solution is to be performed. Three Bulk Data entries are available for selecting the solution frequencies. It is important to remember that each specified frequency results in an independent solution at the specified excitation frequency.

To select the loading frequencies, the FREQ, FREQ1, and FREQ2 Bulk Data entries are used. The FREQ entry defines discrete excitation frequencies. The FREQ1 entry defines a starting frequency F_{start}, a frequency increment Δf, and the number of frequency increments NDF to be solved. The FREQ2 entry defines a starting frequency F_{start}, an ending frequency F_{end}, and the number of logarithmic intervals NF to be used in the frequency range.

All FREQi entries with the same Set ID are used.

The FREQi Bulk Data entries are selected by the FREQUENCY Case Control command. All FREQi entries with the same selected Set ID are applied in the analysis; therefore, any combination of FREQ, FREQ1, and FREQ2 entries can be used.

Examples of FREQi entries are shown. The example FREQ entry specifies 10 specific (unequally spaced) loading frequencies to be performed in the analysis. The FREQ1 example specifies 14 frequencies between 2.9 Hz and 9.4 Hz in increments of 0.5 Hz. The FREQ2 example specifies six logarithmic frequency intervals between 1.0 and 8.0 Hz, resulting in frequencies at 1.0, 1.4142, 2.0, 2.8284, 4.0, 5.6569, and 8.0 Hz being used for the analysis.

The FREQ entry has the following format:

1	2	3	4	5	6	7	8	9	10
$FREQ	SID	F	F	F	F	F	F	F	
$	F	F	F	F	F	F	F	F	
FREQ	3	2.98	3.05	17.9	21.3	25.6	28.8	31.2	
	29.2	22.4	19.3						

Field	Contents
SID	Set ID specified by a FREQUENCY Case Control command.
F	Frequency value (cycles per unit time).

The FREQ1 entry has the following format:

1	2	3	4	5	6	7	8	9	10
$FREQ1	SID	F_{start}	Δf	NDF					
FREQ1	6	2.9	0.5	13					

Field	Contents
SID	Set ID specified by a FREQUENCY Case Control command.
F_{start}	Starting frequency in set (cycles per unit time).
Δf	Frequency increment (cycles per unit time).
NDF	Number of frequency increments.

The FREQ2 entry has the following format:

1	2	3	4	5	6	7	8	9	10
$FREQ2	SID	F_{start}	F_{end}	NF					
FREQ2	9	1.0	8.0	6					

Field	Contents
SID	Set ID specified by a FREQUENCY Case Control command.
F_{start}	Starting frequency (cycles per unit time).
F_{end}	Ending frequency (cycles per unit time).
NF	Number of logarithmic intervals.

The three sets of excitation frequencies are all combined in a single analysis if the Set IDs are identical.

5.6 Frequency Response Considerations

Exciting an undamped (or modal or viscous damped) system at 0.0 Hz using direct frequency response analysis gives the same results as a static analysis and also gives almost the same results when using modal frequency response (depending on the number of retained modes). Therefore, if the maximum excitation frequency is much less than the lowest resonant frequency of the system, a static analysis is probably sufficient.

Undamped or very lightly damped structures exhibit large dynamic responses for excitation frequencies near resonant frequencies. A small change in the model (or running it on another computer) may result in large changes in such responses.

Use a fine enough frequency step size (Δf) to adequately predict peak response. Use at least five points across the half-power bandwidth (which is approximately $2\zeta f_n$ for an SDOF system) as shown in Figure 5-7.

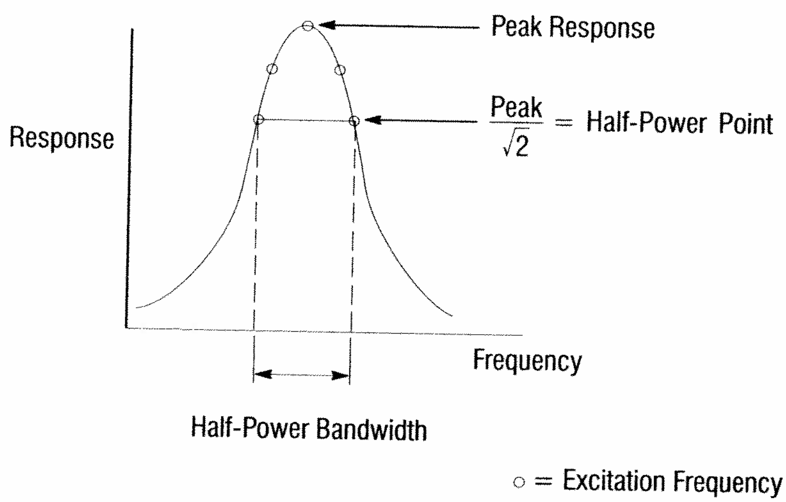

Figure 5-7. Half-Power Bandwidth.

For maximum efficiency, an uneven frequency step size should be used. Smaller frequency spacing should be used in regions near resonant frequencies, and larger frequency step sizes should be used in regions away from resonant frequencies.

NASTRAN FREQUENCY RESPONSE ANALYSIS

Frequency Response = Harmonic Response = Steady State Sine Response
Input = Steady State Sinseusoidal Forcing Function
Output = Steady State Sineusoidal Response (stress/disp/acce)
Note: This does <u>not</u> include transient effects (ie start-up)
Linear System = small displacement, linear material

Dynamics equation:

$$M \ddot{U} + B \dot{U} + K U = P$$

K = stiffeness matrix (from elements, ie cbeam, celas, etc)
B = damping matrix (from cdamp, g on mat1, etc)
M = mass matrix (from mass density on mat1, and lumped conm2)

Assume sineusoidal load

$$P = \mathit{P} e^{i\omega t}$$

At steady state, will get sineusoidal response at same freq as input load.

$$U = \mathit{U} e^{i\omega t} \qquad \dot{U} = i\omega \mathit{U} e^{i\omega t} \qquad \ddot{U} = -\omega^2 \mathit{U} e^{i\omega t}$$

Substitue into dynamics equation

$$[-\omega^2 M + i\omega B + K]\, \mathit{U} e^{i\omega t} = \mathit{P} e^{i\omega t}$$

Cancel

$$[-\omega^2 M + i\omega B + K]\, \mathit{U} = \mathit{P}$$

For any given freq ω, the above reduces to

$D\,\mathit{U} = \mathit{P}$ where $D = -\omega^2 M + i\omega B + K$ (NOTE: $D, \mathit{U}, \mathit{P}$ are <u>complex</u>)

Sove the above at any ω, just like solving K U = P, except complex math

<u>DIRECT FREQUENCY RESPONSE</u> = **SOL 108**

* User specifies model => K,B,M
* User specifies amplitude of load = P on **RLOAD,DAREA,TABLED1**
* User specifies freq list at which want solutions = f on **FREQ** card
* User specifies damping by **CDAMP**, g on **MAT1**
* NASTRAN solves for amplitude of response = $\mathit{U}, \dot{\mathit{U}}, \ddot{\mathit{U}}, \sigma$, etc
* Note: $e^{i\omega t}$ is <u>implied</u>, and not identified in load or response
* Output requests include **DISP,VELO,ACCE,STRESS**
* SORT1 for all response at 1 freq, SORT2 for 1 response at all freq
* To send response to XL, use **param,post,0**can plot contours or XY

Only 1 BC per run (SPC). Subcases are used for multiple load cases

```
$------------------------------------------------------------------------
$                  DIRECT FREQUENCY RESPONSE ANALYSIS
$                  = STEADY STATE HARMONIC RESPONSE
$                  = SINEUSOIDAL INPUT=f=Fsin(wt) AND OUTPUT=u=Usin(wt)
$                  [-Mw2 + iBw +K]U = F  where U=amplitude(u)
$------------------------------------------------------------------------
$
TIME 5                              (DYNAMICS REQUIRE LARGER TIME LIMITS)
SOL 108                             (DIRECT FREQUENCY ANALYSIS)
CEND                                (SEE H/DA SEC 9 FOR LIST OF ALTERS)
TITLE=
SUBTITLE=
SPC=9                               (ONLY 1 BC SET PER RUN)
MPC=11                              (TURN ON MPC EQUATIONS)
SUBCASE 1                           (1 SUBCASE FOR EACH LOAD CONDITION)
LABEL=
DLOAD=20                            (DYNAMIC LOAD: CALL RLOADi or DLOAD)
FREQ=30                             (FREQUENCIES SOLVED FOR: CALL FREQ)
DISP(SORT2,PHASE)=                  (OUTPUT REQUEST FOR DISP AMPLITUDE)
ACCE(SORT2,PHASE)=                  (OUTPUT REQUEST FOR ACCE AMPLITUDE)
STRESS=                             (OUTPUT REQUEST FOR STRESS AMPLITUDE)
SUBCASE 2                           (ADDITIONAL LOAD CASES)
...
BEGIN BULK
GRIDS,ELEMENTS,PROPERTIES...        (MODEL DESCRIPTION)
SPC,9...                            (BC FOR THIS RUN - ONLY 1 SET/RUN ACTIVE)
MPC,11...                           (MPC = LINEAR EQNS, IF DESIRED)
MAT1...                             (INCLUDE MASS DENSITY FOR ALL ELEMENTS)
CONM2...                            (LUMPED MASSES AND INERTIAS)
PARAM,GRDPNT,0                      (PRINT MASS PROPERTY TABLE)
PARAM,POST,0                        (CREATE XL XDB FILE)
...
DLOAD,20,1.,1.,21                   (COMBINES RLOADi CARDS)
RLOAD2,21,22,,,24                   (DEFINE LOAD AMPLITUDE: CALL DAREA,TABLED)
DAREA,22...                         (POINT OF LOAD APLICATION=GRID,COMP,MAG)
TABLEDi,24...                       (LOAD AMPLITUDE AS FUNCTION OF FREQUENCY)
FREQ,30...                          (LIST FREQUENCIES (f) AT WHICH WANT SOLUTION)
...
PARAM,G,0.04                        (STRUCTURAL DAMPING COEFF ( 2% CRITICAL))
CVISC,CDAMP                         (VISCOUS DAMPING ELEMENTS)
ENDDATA
-------------------------------------------------------------------------
$------------------------------------------------------------------------
REF: MSC/Handbook for Dynamic Analysis = HDA

For complete list of damping options HDA Sec 3.2

Note:  In this solution, you describe load amplitude and phase (F)
Results are response amplitude and phase (U). The sin(wt) implied.
```

```
$----------------------------------------------------------------
$ DIRECT FREQUENCY RESPONSE - 1 DOF SPRING/MASS/DAMPER SYSTEM
$ BASE EXCITATION (using large mass technique)
$ Ref: Thomson, Vibrations 3rd Ed, Figure 3.5-1
$----------------------------------------------------------------
$ Note: UNITS MUST BE CONSISTENT
$ Solution 108 is steady-state harmonic response
$     via "direct" approach, NOT modal
$----------------------------------------------------------------
$
time 5
sol 108
cend
title=1 dof spring mass system
subtitle=Thomson Figure 3.5-1
mpc=75                       $turn on mpc
subcase 1
label=ss sine force on grid 50 (base)
dload=80                     $call rload2 (dyn load)
freq=85                      $call freq (f steps)
set 8=50,51,59
disp(sort2,phase)=8          $phase=>magnitude/phase
velo(sort2,phase)=8
acce(sort2,phase)=8
begin bulk
param,post,0                 $output to XL
param,grdpnt,0               $print mass prop table
$GRID POINTS WITH 5 DOF FIXED
grid,50,,0.,0.,0.,,23456
grid,51,,1.,0.,0.,,23456
grid,59,,0.,0.,0.,,23456
$USE MPC TO CALC RELATIVE MOTION
$ -59+51-50=0  => 59=51-50
mpc,75,59,1,-1.,51,1,1.
,,50,1,-1.
$SPRINGS CONNECTING X DISPL
celas1,21,20,50,1,51,1
pelas,20,39.478          $K=(2*pi)**2
$LUMPED MASSES
conm2,31,51,,1.          $M=1
conm2,30,50,,1.+4        $LARGE MASS=BASE
$DAMPER C=B=.25Cc=.25sqrt(K*M)=pi => eta=.25
cdamp1,41,40,50,1,51,1
pdamp,40,3.14159
$APPLIED LOAD
rload2,80,81,,,82
darea,81,50,1,1.+4          $ENFORCE DISP=1
tabled4,82,0.,1.,0.,100.  $ACC=w**2=[(2pi)**2]*f**2
,0.,0.,39.478,endt
freq1,85,0.,.05,60       $freq=Hz, not radians/sec
ENDDATA
```

base.dat

$$m\ddot{x} = -k(x-y) - c(\dot{x}-\dot{y})$$

Making the substitution

$$z = x - y = 59$$

Eq. (3.5-1) becomes

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$$
$$= m\omega^2 Y \sin \omega t$$

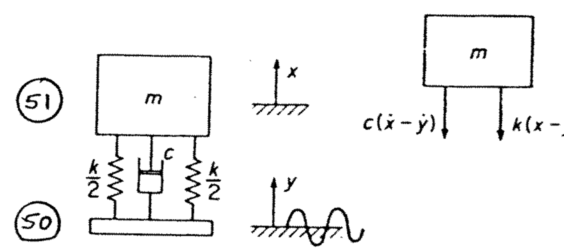

Figure 3.5-1. System excited by motion of support pc

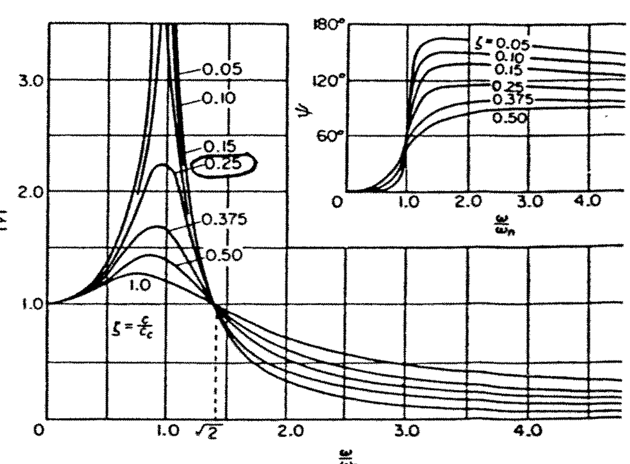

Figure 3.5-2. Plot of Eqs. (3.5-8) and (3.5-9).

```
$------------------------------------------------------------
$ DIRECT FREQUENCY RESPONSE - 2 DOF SPRING/MASS SYSTEM
$ Ref: Thomson, Vibrations 3rd Ed, Example 5.3-1
$------------------------------------------------------------
$ Note: UNITS MUST BE CONSISTENT
$ Solution 108 is steady-state harmonic response
$     via "direct" approach, NOT modal
$------------------------------------------------------------
$
time 5
sol 108                        $direct freq resp
cend
title=2 dof spring mass system
subtitle=Thomson example 5.3-1
spc=70                         $turn on bc
subcase 1
label=ss sine force on grid 51
dload=80                       $call rload2 (dyn load)
freq=85                        $call freq (f steps)
set 8=51,52
disp(sort2,real)=8             $real=>real/imag
$disp(sort2,phase)=8           $phase=>magnitude/phase
velo(sort2,real,plot)=8 $send to XL, don't print
begin bulk
$GRID POINTS WITH 5 DOF FIXED
grid,50,,0.,0.,0.,,23456
grid,51,,1.,0.,0.,,23456
grid,52,,2.,0.,0.,,23456
grid,53,,3.,0.,0.,,23456
$SPRINGS CONNECTING X DISPL
celas1,21,20,50,1,51,1
celas1,22,20,51,1,52,1
celas1,23,20,52,1,53,1
pelas,20,18.      $K=18
$LUMPED MASSES
conm2,31,51,,2.   $M=2
conm2,32,52,,2.
$FIXED BC AT WALLS
spc1,70,1,50,53
$APPLIED LOAD
rload2,80,81,,,82
darea,81,51,1,18.          $force on grid
tabled1,82                 $force vs freq
,0.,1.,1000.,1.,endt
freq1,85,0.,.05,30     $freq=Hz, not radians/sec
$ 1% of critical damping added to prevent infinite displ
$ cvisc = analogous to crod (viscous)
$ cdamp = analogous to celas (viscous)
$ g = multiplies K (structural)
cdamp1,41,40,50,1,51,1
cdamp1,42,40,51,1,52,1
cdamp1,43,40,52,1,53,1
pdamp,40,.12        $b=.01C=.01sqrt(k*m)=.01*2*w*m
ENDDATA
```

dfreq.dat

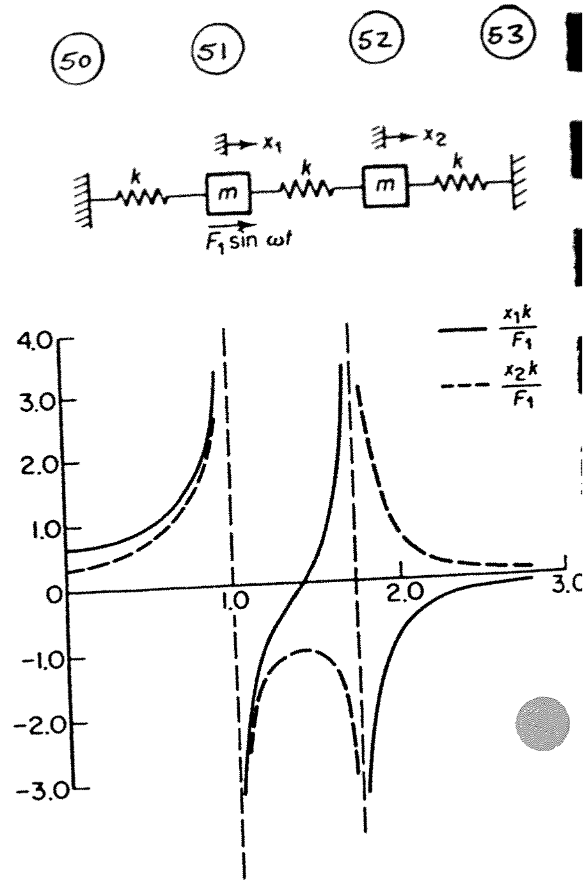

Figure 5.3-2. Forced response of the two-DOF system.