C H A P T E R 6

TRANSIENT RESPONSE ANALYSIS

Transient response analysis is the most general method for computing forced dynamic response. The purpose of a transient response analysis is to compute the behavior of a structure subjected to time-varying excitation. The transient excitation is explicitly defined in the time domain. All of the forces applied to the structure are known at each instant in time. Forces can be in the form of applied forces and/or enforced motions (see Chapter 7).

Transient response analysis is performed in the time domain.

The important results obtained from a transient analysis are typically displacements, velocities, and accelerations of grid points, and forces and stresses in elements.

Depending upon the structure and the nature of the loading, two different numerical methods can be used for a transient response analysis: direct and modal. The direct method performs a numerical integration on the complete coupled equations of motion. The modal method utilizes the mode shapes of the structure to reduce and uncouple the equations of motion (when modal or no damping is used); the solution is then obtained through the summation of the individual modal responses. The choice of the approach is problem dependent. The two methods are described in Sections 6.1 and 6.2.

6.1 Direct Transient Response Analysis

In direct transient response, structural response is computed by solving a set of coupled equations using direct numerical integration. Begin with the dynamic equation of motion in matrix form

$$[M]\{\ddot{u}(t)\} + [B]\{\dot{u}(t)\} + [K]\{u(t)\} = \{P(t)\} \tag{6-1}$$

The fundamental structural response (displacement) is solved at discrete times, typically with a fixed integration time step Δt.

By using a central finite difference representation for the velocity $\{\dot{u}(t)\}$ and the acceleration $\{\ddot{u}(t)\}$ at discrete times,

$$\begin{aligned} \{\dot{u}_n\} &= \frac{1}{2\Delta t}\{u_{n+1} - u_{n-1}\} \\ \{\ddot{u}_n\} &= \frac{1}{\Delta t^2}\{u_{n+1} - 2u_n + u_{n-1}\} \end{aligned} \tag{6-2}$$

and averaging the applied force over three adjacent time points, the equation of motion can be rewritten as:

Same form as
Modified Central Dif
but N Dof

$$\begin{aligned} \left[\frac{M}{\Delta t^2}\right]\left(u_{n+1} - 2u_n + u_{n-1}\right) + \left[\frac{B}{2\Delta t}\right]\left(u_{n+1} - u_{n-1}\right) \\ + \left[\frac{K}{3}\right]\left(u_{n+1} + u_n + u_{n-1}\right) = \frac{1}{3}\left(P_{n+1} + P_n + P_{n-1}\right) \end{aligned} \tag{6-3}$$

Collecting terms, the equation of motion can be rewritten as:

$$[A_1]\{u_{n+1}\} = [A_2] + [A_3]\{u_n\} + [A_4]\{u_{n-1}\} \tag{6-4}$$

where $[A_1] = \left[\frac{M}{\Delta t^2} + \frac{B}{2\Delta t} + \frac{K}{3}\right]$

$$[A_2] = \frac{1}{3}\{P_{n+1} + P_n + P_{n-1}\}$$

$$[A_3] = \left[\frac{2M}{\Delta t^2} - \frac{K}{3}\right]$$

$$[A_4] = \left[-\frac{M}{\Delta t^2} + \frac{B}{2\Delta t} - \frac{K}{3}\right]$$

Matrix $[A_1]$ is termed the dynamic matrix, and $[A_2]$ is the applied force (averaged over three adjacent time points). This approach is similar to the classical Newmark-Beta direct integration method except that $\{P(t)\}$ is averaged over three time points and $[K]$ is modified such that the dynamic equation of motion reduces to a static solution $[K]\{u_n\} = \{P_n\}$ if no $[M]$ or $[B]$ exists.

The transient solution is obtained by decomposing $[A_1]$ and applying it to the right-hand side of the above equation. In this form, the solution behaves like a succession of static solutions with each time step performing a forward-backward substitution (FBS) on a new load vector. Note that the transient nature of the solution is carried through by modifying the applied force matrix $[A_2]$ with the $[A_3]$ and $[A_4]$ terms.

The matrices are constant throughout the analysis.

In its simplest form, the $[M]$, $[B]$, and $[K]$ matrices are assumed to be constant throughout the analysis and do not change with time. Special solution methods are available in MSC/NASTRAN for variations in these matrices (see the *MSC/NASTRAN Advanced Dynamic Analysis User's Guide)*.

A significant benefit presents itself if Δt remains constant during the analysis. With a constant Δt, the $[A_1]$ matrix needs to be decomposed only once. Each progressive step in the analysis is only an FBS of a new load vector. If Δt is changed, $[A_1]$ must be redecomposed, which can be a costly operation in large problems.

Another efficiency in the direct transient solution is that the output time interval may be greater than the solution time interval. In many cases it is not necessary to sample output response at each solution time. For example, if the solution is performed every 0.001 second the results can be output every fifth time step or every 0.005 second. This efficiency reduces the amount of output.

Damping in Direct Transient Response

The damping matrix $[B]$ is used to represent the energy dissipation characteristics of a structure. In the general case, the damping matrix is comprised of several matrices

$$[B] = \left[B^1\right] + \left[B^2\right] + \frac{G}{W_3}[K] + \frac{1}{W_4}\sum G_E\left[K_E\right] \qquad (6\text{-}5)$$

where $\left[B^1\right]$ = damping elements (CVISC, CDAMPi) + B2GG

$\left[B^2\right]$ = B2PP direct input matrix + transfer functions

G = overall structural damping coefficient (PARAM,G)

W_3 = frequency of interest in radians per unit time (PARAM,W3) for the conversion of overall structural damping into equivalent viscous damping

$[K]$ = global stiffness matrix

G_E = element structural damping coefficient (GE on the MATi entry)

W_4 = frequency of interest in radians per unit time (PARAM,W4) for conversion of element structural damping into equivalent viscous damping

$\left[K_E\right]$ = element stiffness matrix

Structural damping is converted to equivalent viscous damping.

Transient response analysis does not permit the use of complex coefficients. Therefore, structural damping is included by means of equivalent viscous damping. To appreciate the impact of this on the solution, a relation between structural damping and equivalent viscous damping must be defined.

The viscous damping force is a damping force that is a function of a damping coefficient b and the velocity. It is an induced force that is represented in the equation of motion using the $[B]$ matrix and velocity vector.

$$[M]\{\ddot{u}(t)\} + [B]\{\dot{u}(t)\} + [K]\{u(t)\} = \{P(t)\} \qquad (6\text{-}6)$$

The structural damping force is a displacement-dependent damping. The structural damping force is a function of a damping coefficient G and a complex component of the structural stiffness matrix.

$$[M]\{\ddot{u}(t)\} + (1 + iG)[K]\{u(t)\} = \{P(t)\} \qquad (6\text{-}7)$$

Assuming constant amplitude oscillatory response for an SDOF system, the two damping forces are identical if

$$Gk = b\omega \tag{6-8}$$

or

$$b = \frac{Gk}{\omega} \tag{6-9}$$

1-D

$$\zeta = \frac{b}{2k/\omega_n} = \frac{b\omega_n}{2k}$$

$G = 2\zeta$ @ $\omega = \omega_n$

Therefore, if structural damping G is to be modeled using equivalent viscous damping b, then the equality Eq. (6-9) holds at only one frequency (see Figure 6-1).

Two parameters are used to convert structural damping to equivalent viscous damping. An overall structural damping coefficient can be applied to the entire system stiffness matrix using PARAM,W3,r where r is the circular frequency at which damping is to be made equivalent. This parameter is used in conjunction with PARAM,G. The default value for W3 is 0.0, which causes the damping related to this source to be ignored in transient analysis.

PARAM,W4 is an alternate parameter used to convert element structural damping to equivalent viscous damping. PARAM,W4,r is used where r is the circular frequency at which damping is to be made equivalent. PARAM,W4 is used in conjunction with the GE field on the MATi entry. The default value for W4 is 0.0 which causes the related damping terms to be ignored in transient analysis.

Units for PARAM,W3 and PARAM,W4 are radians per unit time. The choice of W3 or W4 is typically the dominant frequency at which the damping is active. Often, the first natural frequency is chosen, but isolated individual element damping can occur at different frequencies and can be handled by the appropriate data entries.

Units for W3 and W4 are radians per unit time.

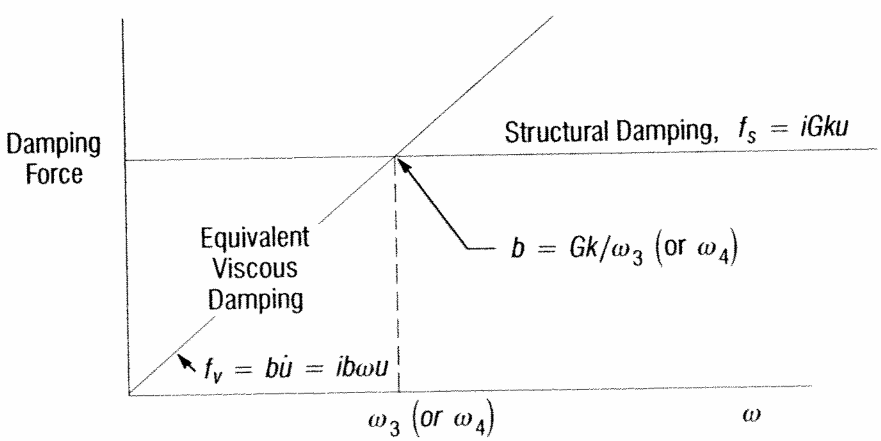

Figure 6-1. Structural Damping Versus Viscous Damping (Constant Oscillatory Displacement).

Initial Conditions in Direct Transient Response

You may impose initial displacements and/or velocities in direct transient response. The TIC Bulk Data entry is used to define initial conditions on the components of grid points. The IC Case Control command is used to select TIC entries from the Bulk Data.

If initial conditions are used, initial conditions should be specified for all DOFs having nonzero values. Initial conditions for any unspecified DOFs are set to zero.

Initial conditions $\{u_0\}$ and $\{\dot{u}_0\}$ are used to determine the values of $\{u_{-1}\}$, $\{P_0\}$, and $\{P_{-1}\}$ used in Eq. (6-4) to calculate $\{u_1\}$.

Alternative to previously discussed IC's.

$$\{u_{-1}\} = \{u_0\} - \{\dot{u}_0\}\,\Delta t \quad (6\text{-}10)$$

$$\{P_{-1}\} = [K]\{u_{-1}\} + [B]\{\dot{u}_0\} \quad (6\text{-}11)$$

In the presence of initial conditions, the applied load specified at $t = 0$ is replaced by

$$\{P_0\} = [K]\{u_0\} + [B]\{\dot{u}_0\} \quad (6\text{-}12)$$

Regardless of the initial conditions specified, the initial acceleration for all points in the structure is assumed to be zero (constant initial velocity).

The format for the TIC entry is

1	2	3	4	5	6	7	8	9	10
TIC	SID	G	C	U0	V0				

Field	Contents
SID	Set ID specified by the IC Case Control command.
G	Grid, scalar, or extra point.
C	Component number.
U0	Initial displacement.
V0	Initial velocity.

Initial conditions may be specified only in direct transient response. In modal transient response all initial conditions are set to zero. Initial conditions may be specified only in the a-set (see Chapter 11).

2 Modal Transient Response Analysis

Modal transient response is an alternate approach to computing the transient response of a structure. This method uses the mode shapes of the structure to reduce the size, uncouple the equations of motion (when modal or no damping is used), and make the numerical integration more efficient. Since the mode shapes are typically computed as part of the characterization of the structure, modal transient response is a natural extension of a normal modes analysis.

As a first step in the formulation, transform the variables from physical coordinates $\{u\}$ to modal coordinates $\{\xi\}$ by

$$\{u(t)\} = [\phi]\{\xi(t)\} \tag{6-13}$$

The mode shapes $[\phi]$ are used to transform the problem in terms of the behavior of the modes as opposed to the behavior of the grid points. Equation (6-13) represents an equality if all modes are used; however, because all modes are rarely used, the equation usually represents an approximation.

To proceed, temporarily ignore the damping, resulting in the equation of motion

$$[M]\{\ddot{u}(t)\} + [K]\{u(t)\} = \{P(t)\} \tag{6-14}$$

If the physical coordinates in terms of the modal coordinates (Eq. (6-13) are substituted into Eq. (6-14)), the following equation is obtained:

$$[M][\phi]\{\ddot{\xi}(t)\} + [K][\phi]\{\xi(t)\} = \{P(t)\} \tag{6-15}$$

This is now the equation of motion in terms of the modal coordinates. At this point, however, the equations remain coupled.

To uncouple the equations, premultiply by $[\phi]^T$ to obtain

$$[\phi]^T[M][\phi]\{\ddot{\xi}\} + [\phi]^T[K][\phi]\{\xi\} = [\phi]^T\{P(t)\} \tag{6-16}$$

where $[\phi]^T[M][\phi]$ = modal (generalized) mass matrix

$[\phi]^T[K][\phi]$ = modal (generalized) stiffness matrix

$[\phi]^T\{P\}$ = modal force vector

The final step uses the orthogonality property of the mode shapes to formulate the equation of motion in terms of the generalized mass and stiffness matrices that are diagonal matrices. These matrices do not have off-diagonal terms that couple the equations of motion. Therefore, in this form, the modal equations of motion are uncoupled. In this uncoupled form, the equations of motion are written as a set on uncoupled SDOF systems as

$$m_i \ddot{\xi}_i(t) + k_i \xi_i(t) = p_i(t) \tag{6-17}$$

where m_i = i-th modal mass

k_i = i-th modal stiffness

p_i = i-th modal force

Note that there is no damping in the resulting equation. The next subsection describes how to include damping in modal transient response.

Once the individual modal responses $\xi_i(t)$ are computed, physical responses are recovered as the summation of the modal responses

$$\{u(t)\} = [\phi]\{\xi(t)\} \tag{6-18}$$

Since numerical integration is applied to the relatively small number of uncoupled equations, there is not as large a computational penalty for changing Δt as there is in direct transient response analysis. However, a constant Δt is still recommended.

Another efficiency option in the modal transient solution is that the output time interval may be greater than the solution time interval. In many cases, it is not necessary to sample output response at each solution time. For example, if the solution is performed every 0.001 second, the results can be output every fifth time step or every 0.005 second. This efficiency reduces the amount of output.

Damping in Modal Transient Response Analysis

If the damping matrix $[B]$ exists, the orthogonality property (see Section 3.2) of the modes does not, in general, diagonalize the generalized damping matrix

$$[\phi]^T[B][\phi] \neq \text{diagonal} \tag{6-19}$$

In the presence of a $[B]$ matrix, the modal transient approach solves the coupled problem in terms of modal coordinates using the direct transient numerical integration approach described in Section 6.1 as follows:

$$[A_1]\{\xi_{n+1}\} = [A_2] + [A_3]\{\xi_n\} + [A_4]\{\xi_{n-1}\} \tag{6-20}$$

where $[A_1] = [\phi]^T\left[\frac{M}{\Delta t^2} + \frac{B}{2\Delta t} + \frac{K}{3}\right][\phi]$

$$[A_2] = \frac{1}{3}[\phi]^T\{P_{n+1} + P_n + P_{n-1}\}$$

$$[A_3] = [\phi]^T\left[\frac{2M}{\Delta t^2} - \frac{K}{3}\right][\phi]$$

$$[A_4] = [\phi]^T\left[-\frac{M}{\Delta t^2} + \frac{B}{2\Delta t} - \frac{K}{3}\right][\phi]$$

These equations are similar to the direct transient method except that they are in terms of modal coordinates. Since the number of modes used in a solution is typically much less than the number of physical variables, the direct integration of the modal equations is not as costly as with physical variables.

If damping is applied to each mode separately, the decoupled equations of motion can be maintained. When modal damping is used, each mode has damping b_i. The equations of motion remain uncoupled and have the following form for each mode:

$$m_i\ddot{\xi}_i(t) + b_i\dot{\xi}_i(t) + k_i\,\xi_i(t) = p_i(t) \tag{6-21}$$

or

$$\ddot{\xi}_i(t) + 2\zeta_i\omega_i\dot{\xi}_i(t) + \omega_i^2\xi_i(t) = \frac{1}{m_i}p_i(t) \tag{6-22}$$

where $\zeta_i = b_i/(2m_i\omega_i) \equiv$ modal damping ratio

$\omega_i^2 = k_i/m_i \equiv$ modal frequency (eigenvalue)

The TABDMP1 Bulk Data entry defines the modal damping ratios. A table is created by the frequency-damping pairs specified on a TABDMP1 entry. The solution refers to this table for the damping value to be applied at a particular frequency. The TABDMP1 Bulk Data entry has a Set ID. A particular TABDMP1 table is activated by selecting the Set ID with SDAMPING = Set ID Case Control command.

1	2	3	4	5	6	7	8	9	10
TABDMP1	ID	TYPE							
	f_1	g_1	f_2	g_2	f_3	g_3	-etc.-	ENDT	

Field	Contents
TID	Table identification number.
TYPE	Type of damping units: G (default) CRIT Q
fi	Frequency value (cycles per unit time).
gi	Damping value in the units specified.

At resonance, the three types of damping are related by the following equations:

$$
\begin{aligned}
\zeta_i &= \frac{b_i}{b_{cr}} = \frac{G_i}{2} \\
b_{cr} &= 2m_i\,\omega_i \\
Q_i &= \frac{1}{2\zeta_i} = \frac{1}{G_i}
\end{aligned}
\qquad (6\text{-}23)
$$

The values of *fi* (units = cycles per unit time) and *gi* define pairs of frequencies and dampings. Note that *gi* can be entered as structural damping (default), critical damping, or quality factor. The entered damping is internally converted to structural damping using Eq. (6-23). Straight-line interpolation is used for modal frequencies between consecutive *fi* values. Linear extrapolation is used at the ends of the table. ENDT ends the table input.

For example, if modal damping is entered using Table 6-1 and if modes exist at 1.0, 2.5, 3.6, and 5.5 Hz, MSC/NASTRAN interpolates and extrapolates as shown in Figure 6-2 and the table. Note that there is no table entry at 1.0 Hz; MSC/NASTRAN uses the first two table entries at $f = 2.0$ and $f = 3.0$ to extrapolate the value for $f = 1.0$.

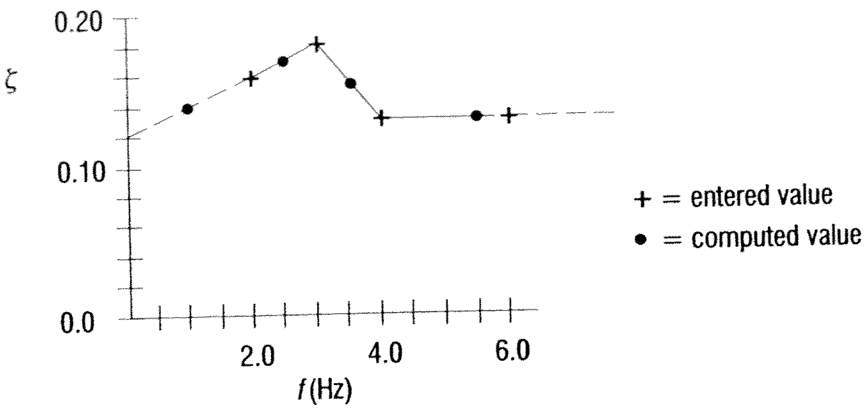

Figure 6-2. Example TABDMP1.

Table 6-1. Example TABDMP1 Interpolation/Extrapolation.

Entered		Computed	
f	ζ	f	ζ
2.0	0.16	1.0	0.14
3.0	0.18	2.5	0.17
4.0	0.13	3.6	0.15
6.0	0.13	5.5	0.13

1	2	3	4	5	6	7	8	9	10
TABDMP1	10	CRIT							+TAB1
+TAB1	2.0	0.16	3.0	0.18	4.0	0.13	6.0	0.13	+TAB2
+TAB2	ENDT								

With the modal equations in the form of Eq. (6-22), an efficient uncoupled analytical integration algorithm is used to solve for modal response as decoupled SDOF systems. Each of the modal responses is computed using

0, No initial conditions for modal transient response.

$$\xi(t) = e^{-bt/2m}\left(\xi_0 \cos \omega_d\, t + \cancel{\frac{\dot{\xi}_0 + (b/2m)\xi_0}{\omega_d} \sin \omega_d t}\right)$$

$$+ e^{-bt/2m}\frac{1}{m\omega_d}\int_0^t e^{b\tau/2m} p(\tau) \sin \omega_d\, (t-\tau)d\tau \qquad (6\text{-}24)$$

In a modal transient analysis, you may add nonmodal damping (CVISC, CDAMPi, GE on the MATi entry, or PARAM,G). With nonmodal damping, there is a computational penalty due to the coupled $[B]$ matrix, causing the coupled solution algorithm to be used. In modal transient response analysis, it is recommended that you use only modal damping (TABDMP1). If discrete damping is desired, direct transient response analysis is recommended.

Initial conditions cannot be used for modal transient response analysis.

Note that there are no nonzero initial conditions for modal transient response analysis.

Mode Truncation in Modal Transient Response Analysis

It is possible that not all of the computed modes are required in the transient response solution. Often, only the lowest few suffice for dynamic response calculation. It is quite common to evaluate the frequency content of transient loads and determine a frequency above which no modes are noticeably excited. This frequency is called the cutoff frequency. The act of specifically not using all of the modes of a system in the solution is termed mode truncation. Mode truncation assumes that an accurate solution can be obtained using a reduced set of modes. The number of modes used in a solution is controlled in a modal transient response analysis through a number of methods.

The frequency range selected on the eigenvalue entry (EIGRL or EIGR) is one means to control the frequency range used in the transient response solution. Also, three parameters are available to limit the number of modes included in the solution. PARAM,LFREQ gives the lower limit on the frequency range of retained modes, and PARAM,HFREQ gives the upper limit on the frequency range of retained modes. PARAM,LMODES gives the number of the lowest modes to be retained. These parameters can be used to include the desired set of modes. Note that the default is for all computed modes to be retained.

It is very important to remember that truncating modes in a particular frequency range may truncate a significant portion of the behavior in that frequency range. Typically, high-frequency modes are truncated because they are more costly to compute. So, truncating high-frequency modes truncates high frequency response. In most cases, high-frequency mode truncation is not of concern. You should evaluate the truncation in terms of the loading frequency and the important characteristic frequencies of the structure.

Dynamic Data Recovery in Modal Transient Response Analysis

In modal transient response analysis, two options are available for recovering displacements and stresses: mode displacement method and matrix method. Both methods give the same answers, although with cost differences.

The mode displacement method computes the total physical displacements for each time step from the modal displacements and then computes element stresses from the total physical displacements. The number of operations is proportional to the number of time steps.

The matrix method computes displacements per mode and element stresses per mode, and then computes physical displacements and element stresses as the summation of modal displacements and element stresses. Costly operations are proportional to the number of modes.

Since the number of modes is usually much less that the number of time steps, the matrix method is usually more efficient and is the default. The mode displacement method can be selected by using PARAM,DDRMM,-1 in the Bulk Data. The mode displacement method is required when "time frozen" deformed structure plots are requested (see Chapter 9).

The mode acceleration method (Chapter 11) is another form of data recovery for modal transient response analysis. This method can provide better accuracy since detailed local stresses and forces are subject to mode truncation and may not be as accurate as the results computed with the direct method.

6.3 Modal Versus Direct Transient Response

Some general guidelines can be used in selecting modal transient response analysis versus direct transient response analysis. These guidelines are summarized in Table 6-2.

Table 6-2. Modal Versus Direct Transient Response.

	Modal	Direct
Small Model		X
Large Model	X	
Few Time Steps		X
Many Time Steps	X	
High Frequency Excitation		X
Normal Damping		X
Higher Accuracy		X
Initial Conditions		X

In general, larger models may be solved more efficiently in modal transient response because the numerical solution is a solution of a smaller system of uncoupled equations. This result is certainly true if the natural frequencies and mode shape were computed during a previous stage of the analysis. Using Duhamel's integral (see Reference 5 in Appendix M.1) to solve the uncoupled equations is very efficient even for very long duration transients. On the other hand, the major portion of the effort in a modal transient response analysis is the calculation of the modes. For large systems with a large number of modes, this operation can be as costly as direct integration. This is especially true for high-frequency excitation. To capture high frequency response in a modal solution, less accurate high-frequency modes must be computed. For small models with a few time steps, the direct method may be the most efficient because it solves the equations without first computing the modes. The direct method is more accurate than the modal method because the direct method is not concerned with mode truncation. For systems with initial conditions, direct transient response is the only choice.

Table 6-2 provides a starting place for evaluating which method to use. Many additional factors may be involved in the choice of a method, such as contractual obligations or local standards of practice.