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Hydrodynamic relations for direct-drive fast-ignition and conventional inertial confinement fusion implosions

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Relations between stagnation and in-flight phases are derived both analytically and numerically, for hydrodynamic variables relevant to direct-drive inertial confinement fusion implosions. Scaling laws are derived for the stagnation values of the shell density and areal density and for the hot-spot pressure, temperature, and areal density. A simple formula is also derived for the thermonuclear energy gain and in-flight aspect ratio. Implosions of cryogenic deuterium-tritium capsules driven by UV laser energies ranging from 25 kJ to 2 MJ are simulated with a one-dimensional hydrodynamics code to generate the implosion database used in the scaling law derivation. These scaling laws provide guidelines for optimized fuel assembly and laser pulse design for direct-drive fast ignition and conventional inertial confinement fusion. © 2007 American Institute of Physics. [DOI: 10.1063/1.2746812]

I. INTRODUCTION

Inertial confinement fusion (ICF) is an approach to generate fusion energy that relies on the inertia of the fuel mass to provide confinement. The confined fuel must reach high temperatures and densities to produce enough thermonuclear reactions, so that the total energy released by the fusion reactions is much greater than the driver energy required to compress the fuel. In direct-drive ICF, a cryogenic deuterium and tritium (DT) spherical capsule filled with DT gas is accelerated inward by direct laser irradiation. A small portion of the fuel in the center is heated by the pdV work of the imploding shell to reach ignition conditions. This central part of the fuel, which is a low-density plasma, forms a hot spot that initiates a burn wave igniting the cold and dense surrounding shell. In order to compress the fuel to the desired temperature and density, the laser pulse starts by driving a shock through the shell. As the shock propagates inside the shell, the laser power begins to rise and subsequently launches a compression wave traveling inward. Since the shock velocity is always subsonic with respect to the upstream shocked material, this compression wave catches up with the shock right at the shell inner surface, providing the laser pulse timing is correct. When the shock and compression wave merge and break out on the inner shell surface, an outward traveling rarefaction wave is launched from this surface due to the density discontinuity at the shell and gas interface. Before the rarefaction wave reaches the shell outer surface, the latter moves at approximately constant velocity. Once the rarefaction wave breaks out, the shell outer surface senses the lower pressure and accelerates inward under the pressure of the laser driver. This marks the onset of the acceleration phase, during which the shell accelerates to a high implosion velocity. The acceleration phase ends when the laser is turned off and the shell starts traveling at an approximately constant implosion velocity \( V_i \). After the laser is turned off, the shell begins to coast at constant velocity until it slows down due to the pressure build-up in the center. This process is denoted as the deceleration phase.

The deceleration phase starts when the inward traveling shock reflects off the center of the capsule, hits the incoming inner shell surface, and the shell velocity slows down. The low-density gas enclosed by the inner shell surface develops a fairly uniform pressure and becomes part of the hot spot. At this point, the imploding shell acts like a spherical piston on the hot spot until it finally reaches stagnation. The hot-spot pressure and temperature keep increasing as the shell kinetic energy is converted into internal energy through pdV compression work. The hot-spot mass increases because the heat conducted from the hot spot to the shell causes more shell material to ablate off the shell inner surface into the hot spot. When the shell stagnates, the pressure is almost constant throughout the hot spot and the shell. Since the density is much larger in the shell than in the hot spot, the latter has much higher temperature. If the hot spot reaches the ignition conditions (Ref. 1, p. 32; Ref. 2, p. 3952), deuterium-tritium (DT) reactions are self-sustained and generate a burn wave into the shell, thus igniting the main fuel. The values of the hydrodynamic variables, such as entropy, pressure, and velocity, during the acceleration phase are usually referred to as in-flight quantities, and are usually determined by the target characteristics and laser pulse specifications. Since the capsule performance depends on the stagnation parameters such as density, areal density and temperature, it is crucial to determine the relations between the in-flight and the stagnation variables, so that the target and laser pulse can be properly designed to meet the requirements for high performance implosions.

While achieving a high hot-spot temperature is crucial for conventional direct-drive ignition, the fuel assembly for fast ignition (FI) requires a low temperature hot spot surrounded by a massive cold shell of densities within a 300–500 g/cm\(^3\) range, and high areal density. Fast ignition separates the fuel compression from ignition. The precom-
pressed fuel assembly is ignited by a separate external trigger, such as the energetic particle beams generated by a high-intensity short-pulse laser. The fast ignition scheme requires values of the core density within an upper and lower bound imposed by the short-pulse laser characteristics, such as energy, power, and intensity. It also requires a massive core with high areal density to slow down the ignitor particle beam and improve the thermonuclear gain. High temperature in the hot spot is not important, if not detrimental, to fast ignition. Instead, a small and relatively cold hot spot is preferred in that most part of the driver energy is used to compress the fuel assembly rather than heating the hot spot. The hydrodynamic laws relating in-flight and stagnation variables are also helpful to establish the basic design principles to achieve the high densities and high areal densities required in a fast ignition fuel assembly.

In this article, we present a set of hydrodynamic relations between the in-flight and stagnation variables, and discuss the general guidelines for the design of high performance direct-drive conventional ICF and FI implosions. Section II briefly introduces the numerical database used in the derivation and benchmarking of the scaling laws. Section III presents the gain formula and hydrodynamic efficiency based on the “rocket effect” model. The analytic scaling laws for the shell quantities are derived in Sec. IV. In Sec. V, the hot-spot scaling relations are developed from an isobaric model.

II. IMPLOSION SIMULATION DATABASE

The one-dimensional Lagrangian radiation-hydrodynamics code LILAC is used to generate an implosion simulation database for producing numerical fits of various scaling laws. This hydrocode is routinely used for ICF target design studies at the Laboratory for Laser Energetics. It includes SESAME equation-of-state tables, flux-limited Spitzer thermal conduction (the value of the flux limiter is set at $f=0.06$), multigroup radiation transport, multigroup alpha particle transport, and three-dimensional (3-D) laser ray tracing. In each simulation, the thermonuclear burn is turned off, since only the stagnating fuel assembly is studied and the scaling laws are derived by neglecting alpha particle heating. The targets used in the simulations are mostly spherical shells consisting of a single DT ice layer or two layers of wetted-foam $([\text{DT}]_3\text{CH})$ and pure DT ice. All targets are filled with 1 atm DT gas, and the initial aspect ratio of the target varies from 1.0 to 5.5. The relaxation (RX) adiabat shaping technique is used to design most of the laser pulse shapes for these implosions. Typical pulse shapes and target geometry used in the simulations are shown in Fig. 1. The relaxation (RX) laser pulse consists of a prepulse followed by an interval of laser shut-off and the main pulse. Such a laser pulse is used to shape the adiabat in the ablator. The adiabat is usually defined as the ratio of the plasma pressure $P$ to the Fermi pressure $P_F$ of a degenerate electron gas. For a fully ionized DT plasma, the adiabat is approximately $\alpha = P/2.2P_F^{5/3}$, where the plasma pressure $P$ is in megabars and the plasma density $\rho$ is in g/cm$^3$. The RX laser pulse is designed to induce an in-flight shaped adiabat profile that is monotonically decreasing from the ablation front (outer shell surface) where the adiabat is high ($\alpha_{\text{out}} = \alpha_{\text{in}}$), to the inner shell surface where the adiabat is low ($\alpha_{\text{in}} = \alpha_{\text{out}} = \alpha_{\text{init}}$). Shaped adiabat profiles exhibit significant advantages (see Refs. 8–11 for details) with respect to flat adiabat profiles where the adiabat is uniform through the ablator ($\alpha_{\text{out}} = \alpha_{\text{init}} = \alpha_{\text{in}}$). Since the characteristics of the final fuel assembly depend only on the inner surface adiabat $\alpha_{\text{in}}$, the comparison between shaped adiabat and flat adiabat profiles needs to be performed for equal $\alpha_{\text{in}}$. In this case, shaped profiles have larger mass-averaged adiabat ($\alpha_{\text{avg}}$) and larger ablation front adiabats $\alpha_{\text{out}}$ than the corresponding flat adiabat profiles. This causes higher ablation velocities and thicker targets leading to better hydrodynamic stability during the acceleration phase in shaped adiabat implosions. Furthermore, laser pulses using the RX method are easier to implement since they require a lower laser-power contrast ratio (ratio of peak power to foot power) than corresponding flat adiabat pulses.

The implosions of about 50 targets are simulated to fill the database. In these numerical simulations, the driver energy varies from 25 kJ to 2 MJ, adiabat from 0.7 to 4.0, implosion velocity from (1.5 to 5.0) $\times 10^7$ cm/s. Most of the implosions are driven by a UV laser driver with a wavelength $\lambda_L=0.35$ $\mu$m and the laser intensity is adjusted to achieve the best performances. Since the intensity varies over a relatively narrow range $(0.75 \times 10^{15} < I_{15} < 1.05 \times 10^{15}$ W/cm$^2$), it does not represent a good scaling parameter. Thus, all the intensity scaling dependence uses analytic results in the derivation of the scaling laws in later sections.

III. HYDRODYNAMIC EFFICIENCY AND GAIN FORMULAS

In direct-drive ICF, the laser energy is absorbed in a narrow region near the critical surface via inverse bremsstrahlung and the outer layer of the shell is heated by the electron thermal conduction from the critical surface. As this layer heats up, some portion of it gets ablated from the shell. The ablated material expands outward, driving the inner part of the shell inward due to momentum conservation. This is the “rocket effect” that provides the ablation pressure for ICF implosions. In this process, only a small fraction of the deposited laser energy is converted into the shell kinetic en-
energy. The ratio between the shell kinetic energy and the laser energy on target is defined as the hydrodynamic efficiency $\eta$. Since most of the absorbed laser energy is converted into either kinetic energy of the shell or exhaust energy of the ablated material, the hydrodynamic efficiency only depends on the ratio between ablated mass and initial mass. Considering a shell of initial mass $M_0$, final mass $M_f$, ablated mass $M_a = M_0 - M_f$, the hydrodynamic efficiency $\eta$ (Ref. 1, p. 233) can be written as

$$\eta = \frac{E_k}{E_L} = \frac{\left(1 - \frac{M_a}{M_0}\right)\ln\left(1 - \frac{M_a}{M_0}\right)}{M_f/M_0}. \quad (1)$$

In typical direct-drive ICF implosions, the ablated mass does not exceed 80% of the initial mass ($M_a/M_0 < 0.8$). Within this range, Eq. (1) can be approximated by the power law $\eta \sim (M_f/M_0)^{0.87}$. For a shell of inner radius $R$ and in-flight thickness $\Delta_{if}$, the total initial shell mass scales as $M_0 \sim \rho_0 R^2 \Delta_{if}$, where $\rho_0$ is the in-flight shell density. By defining the shell in-flight aspect ratio as $\text{IFAR} = R/\Delta_{if}$, an asymptotic scaling relation between IFAR and the in-flight Mach number $M_{\text{if}}$ is obtained analytically (Ref. 2, p. 3960) for spherical implosions, $\text{IFAR} \sim M_{\text{if}}^2$. Thus, the initial shell mass can be approximated by $M_0 \sim \rho_0 R^2/\text{IFAR} \sim \rho_0 R^2 M_{\text{if}}^2$. The mass ablation rate $M_a$ is proportional to the ablation velocity $V_{\text{a}}$, the in-flight shell density $\rho_{\text{if}}$, and the ablation surface $-R^2$, and scales as $M_a \sim \rho_{\text{if}} V_{\text{a}} R^2$. For a shell of radius $R$ and implosion velocity $V_i$, the total time of acceleration scales with the implosion time $t_i = R/V_i$. Thus, the total mass ablated during the implosion is $M_a \sim M_i V_i^2/\rho_i R^2 V_{\text{a}}$. The ablation velocity, i.e., the speed of the ablation front moving through the shell, is determined by the properties of both the shell and the laser. It is usually expressed in terms of the shell in-flight adiabat $\alpha_{\text{if}}$, laser intensity $I_\lambda$ and wavelength $\lambda$, as $V_{\text{a}} \sim \alpha_{\text{if}} I_\lambda^{1/2} \lambda^{-3/4}$. Using the relation $\alpha_{\text{if}} \sim \rho_{\text{if}}/\rho_i$, and $\rho_{\text{if}} \sim P_{\text{if}}/(I_1/\lambda)^{2/3}$ (Ref. 2, p. 3959), where $P_{\text{if}}$ is the laser-driven ablation pressure, the in-flight Mach number is expressed as

$$M_{\text{if}} \sim \frac{V_i}{\sqrt{\rho_{\text{if}}/\rho_i}} \sim V_i \alpha_{\text{if}}^{-3/10} I_{\text{if}}^{2/15} \lambda_{\text{if}}^{2/15}. \quad (2)$$

Notice that the dependence of the ablation velocity and pressure on the laser parameters comes from the planar scaling. Thus, one can easily derive the ratio between ablated mass and initial mass as $M_a/M_0 = V_{\text{a}} R_{\text{if}}^{1/3} / L_{\text{if}}^{2/3}$. Substituting this relation into equation (1), yields the hydrodynamic efficiency scaling law

$$\eta \sim I_{15}^{0.29} V_i^{0.87} \lambda_{15}^{-0.58}. \quad (3)$$

A fitting formula obtained from the implosion simulation database, which is discussed in Sec. II, leads to a very similar result.

![FIG. 2. (Color online) Hydrodynamic efficiency $\eta$ from simulations (dots) compared to the numerical fit in Eq. (4) (solid line).](image)

$$\eta_{\text{fit}} = \frac{0.051}{I_{15}^{0.25}} \frac{V_i^{0.75}}{3 \times 10^7} \left(\frac{0.35}{\lambda L}\right)^{0.5}, \quad (4)$$

where $I_{15}$ represents the laser intensity in units of $10^{15}$ W/cm$^2$, $V_i$ is the implosion velocity in cm/s, and $\lambda L$ is the laser wavelength in $\mu$m. The coefficient in (4) is adjusted to account for the fraction of absorbed laser energy. The numerical fit (4) is approximately equal to the scaling in Ref. 4 and is compared with the simulation results in Fig. 2. Equation (4) shows that for a fixed laser intensity, $\eta$ increases with higher implosion velocities. This is because fast targets have less initial mass and a greater fraction is ablated off.

The hydrodynamic efficiency in Eq. (4) is now used to derive a scaling relation for the thermonuclear gain of ignited capsules. The thermonuclear energy gain $G$ is defined as the ratio between the thermonuclear energy yield $E_g$ and the energy on target $E_L$. The thermonuclear energy yield comes from the energy of the fusion products. Each D+T fusion reaction generates $E_g = 17.6$ MeV of energy. The total energy output for an ignited shell of final mass $M_f$, with burn-up fraction $\theta = \rho R/(7 + \rho R)$ (Ref. 1, p. 40) is $E_g = M_f \theta E_i/2m_i$, where $m_i = 2.5m_H$ is the average ion mass. The laser energy is written in terms of the shell kinetic energy $E_k = M_f V_{\text{if}}^{2/3}$. The energy gain is expressed as $G = \eta \theta E_i / m_i V_{\text{if}}^2$. Substituting the hydrodynamic efficiency (4) derived above, the energy gain can be rewritten in the following form:

$$G \approx \frac{73}{I_{15}^{0.25}} \left(\frac{3 \times 10^7}{V_i}\right)^{1.25} \left(\frac{\theta R}{0.2}\right)^{0.35} \left(\frac{\lambda}{\lambda_{15}}\right)^{0.5}. \quad (5)$$

Equation (5) shows that the energy gain decreases with the implosion velocity and increases with $\rho R$. For a given driver energy on target, lower implosion velocities require more massive targets, and therefore more fuel available for reactions. Higher $\rho R$ lead to longer confinement time and therefore higher burn fractions. Thus, low implosion velocities...
and higher areal densities are necessary to achieve high gains. The scaling relation for the areal density is derived in Sec. IV. It is important to emphasize that the gain formula in (5) assumes that ignition has occurred and a burn wave has propagated through the dense fuel. In conventional ICF, ignition requires a large enough implosion velocity since the minimum kinetic energy required for ignition scales as $E_{\text{kin}}^\text{ign} \sim \alpha_i^{1/9} V_i^{5/9}$. It follows that hot-spot ignition sets a lower bound on the implosion velocity, thus limiting the maximum gain. Even in fast ignition, the upper bound for the gain is set by a lower limit in the implosion velocity. The minimum implosion velocity is set by the density required for fast ignition as shown by the density scaling in Sec. IV and by the ignition energy scaling. However, the minimum implosion velocity required for ignition is significantly less in FI as compared to conventional ICF, thus leading to higher FI gains for the same driver energy.

IV. STAGNATION SHELL PROPERTIES

A. Density and areal density

For inertial fusion energy (IFE) applications, the thermonuclear gain must be greater than 100 and the areal density must be at least $\sim 3 \text{ g/cm}^2$. To facilitate the pulse design for ICF implosion capsules, it is useful to develop analytic relations for the implosion characteristics. Let us first consider a one-dimensional (1-D) spherical shell at stagnation, as shown in Fig. 3, consisting of a sphere with a central hot spot surrounded by a shell of highly compressed fuel. The cold shell with density $\rho_s$ and temperature $T_s$ has a thickness $\Delta_s$. The hot-spot radius is denoted by $R_h$, which is also regarded as the inner-shell radius. Thus, the outer-shell radius can be written as $R_s = R_h + \Delta_s$. To derive simple scaling relations for the hydrodynamic variables at stagnation, we can adopt the isobaric model, and assume that the stagnation pressure is approximately uniform throughout the hot spot and the cold shell ($p = p_h = p_s$). We denote with $\rho_s$ and $T_h$ the hot-spot density and temperature, respectively, and recognize that, in the isobaric configuration, the denser fuel is much colder than the hot spot, thus leading to $p_h < p_s$ and $T_s > T_h$.

By denoting with $A_s = R_h/\Delta_s$, the shell stagnation aspect ratio, the shell stagnation volume can be written as $V_s = 4\pi R_h^2 \Sigma(A_s)/A_s$, where $\Sigma(x) = 1 + (1/x) + 1/(3x^2)$ is a volume factor. Using the total shell mass at stagnation $M_s$, the compressed shell areal density scales as $\rho_s \Delta_s \sim M_s \Delta_s / V_s \sim M_s / R_h^2 \Sigma(A_s)$. For a shell with implosion velocity $V_i$ and kinetic energy $E_k = M_s V_i^2 / 2$, the shell areal density scaling can be written as

$$\rho_s \Delta_s \sim \frac{E_k}{R_h^2 \Sigma(A_s)}.$$  \hspace{1cm} (6)

Since the shell kinetic energy is converted mostly into internal energy at stagnation, then

$$E_k \sim \rho_s (R_h + \Delta_s)^3.$$ \hspace{1cm} (7)

By setting $\rho_s \sim \alpha_i \rho_s^{4/3}$, where $\alpha_i$ is the stagnation adiabat, the energy conversion condition (7) can be rewritten as $E_k \sim \alpha_i \rho_s^{4/3} (1 + A_s)^3 \Delta_s \sim \alpha_i (p_s \Delta_s)^{4/3} (1 + A_s)^{3/2}$, leading to

$$\Delta_s \sim \frac{E_k^{3/4}}{\alpha_i^{3/4} (p_s \Delta_s)^{5/4} (1 + A_s)^{9/4}}.$$ \hspace{1cm} (8)

Substituting Eq. (8) into (6), and eliminating $R_h = A_s \Delta_s$, yields the following scaling relation for the areal density

$$\rho_s \Delta_s \sim \Phi(A_s) \alpha_i^{-1} E_k^{1/3} V_i^{4/3},$$ \hspace{1cm} (9)

where the function $\Phi$ is defined as $\Phi(x) = (x^2 + x + 1/3)^{2/3}/(1 + x)^{3}$. Using Eq. (8), the shell stagnation thickness can be written as

$$\Delta_s \sim \frac{E_k^{1/3}}{\alpha_i^{1/3} \rho_s^{5/6} (1 + A_s)^{1/6}}.$$ \hspace{1cm} (10)

Substituting Eq. (10) into (9), and solving for $\rho_s$, yields the density scaling relation

$$\rho_s \sim \Psi(A_s) V_i^{3} \alpha_i^{-3/2},$$ \hspace{1cm} (11)

where the function $\Psi$ is defined as $\Psi(x) = [(1 + x)\Phi(x)]^{3/4}$. Notice that $\alpha_i$ and $A_s$ are stagnation variables. The stagnation adiabat is significantly greater than the adiabat of the fuel in flight, in that the return shock driven by the high pressure inside the hot spot increases the fuel adiabat when it propagates outwards through the shell. The compressible thick shell model is used in Ref. 15 to relate the stagnation to the in-flight adiabat leading to $\alpha_i \sim \alpha_{\text{if}} \text{Mach}_{\text{if}}^{2/3}$. A similar relation was found in Ref. 16, and with a slightly different power index in Ref. 17. Using Eq. (2) as the scaling relation for the in-flight Mach number, yields the scaling relation for the stagnation adiabat
Hydrodynamic efficiency, i.e., the return shock. These two scaling relations
are the results of mass and energy conservation, and entropy variation due to
the return shock.

The shell stagnation aspect ratio \( A_s \) is determined numerically by fitting the implosion simulation database described in Sec. II. The simulations show that the stagnation aspect ratio depends primarily on the implosion velocity and weakly on the adiabat. A fit of the simulation results leads to

\[
A_s \sim \alpha_d^{4/5} V_i^{2/3} I_L^{1/15} n_{3/4} L_{1/4},
\]

(12)

Both the density and areal density scaling equations (11) and
(9) can be rewritten in terms of the in-flight variables, leading to

\[
\rho_s \sim \Psi(A_s) \alpha_d^{-6/5} V_i^{2/15} n_{-2/15} L_{-2/15},
\]

(13)

\[
\rho_s A_s \sim \Phi(A_s) \alpha_d^{-4/5} E_k^{1/15} V_i^{4/45} n_{4/45} L_{-4/45}.
\]

(14)

These two scaling relations (14) and (13) are the results of
mass and energy conservation, and entropy variation due to
the return shock.

The shell stagnation aspect ratio \( A_s \) is determined numerically by fitting the implosion simulation database described in Sec. II. The simulations show that the stagnation aspect ratio depends primarily on the implosion velocity and weakly on the adiabat. A fit of the simulation results leads to the following scaling:

\[
A_s^{\text{fit}} = \frac{1.48}{\alpha_d^{0.19}} \left( \frac{V_i}{3 \times 10^7} \right)^{0.96},
\]

(15)

as shown in Fig. 4. This numerical fit is then used in the derivation of the stagnation scaling laws that follows. In typical ICF implosions, the stagnation aspect ratio value falls in the range of \( 1 < A_s < 4 \). Within this interval, the functions \( \Phi(A_s) \) and \( \Psi(A_s) \) can be approximated with the following power laws: \( \Phi(A_s) \sim 1/A_s^{0.34} \) and \( \Psi(A_s) \sim 1/A_s^{0.62} \). Since the kinetic energy \( E_k \) is related to the laser energy \( E_L \) through the hydrodynamic efficiency, i.e., \( E_k = E_L \eta \), substituting (4) and (15) into (14) and (13), yields the following scaling laws for the density and areal density:

\[
\rho_s \sim \alpha_d^{1.08} V_i^{4/15} I_L^{1/15} n_{3/4} L_{-2/15},
\]

(16)

Notice that in the areal density scaling, the laser intensity
dependence is neglected due to the small exponent \( n_{-2/15} \). The dependence of \( \rho_s A_s \) on the implosion velocity is also very weak, only \( V_i^{4/15} \). This is because high compression requires large implosion velocities, and large implosion velocities are obtained, for fixed laser energy, by reducing the mass of the target. Since the areal density needs both compression and fuel mass, these two effects balance each other, making the areal density approximately independent of the implosion velocity.

In order to validate the analytic scaling laws, the shell
stagnation properties are also determined using the implosion database. The shell density is defined as the peak value of the averaged density, with the average carried out over the areal database. The shell density \( \rho_s \) is taken as the maximum value of \( f_s d^3 r \) during the implosion. As shown in Figs. 5 and 6, these quantities are compared favorably with the analytical scaling laws (16) and
(17) and the results in Ref. 4. In the implosion simulation database \( \alpha_{\text{inm}} \) represents the value of the in-flight adiabat on the inner shell surface corresponding to the minimum value of the in-flight adiabat for both shaped and flat adiabat implosions. Notice that the analytic wavelength scalings are used in the numerical fit, since the wavelength is fixed at \( \lambda_L = 35 \mu \text{m} \) in the implosion simulation database. However,
the analytical wavelength scaling is verified by simulating three 100 kJ targets imploded with velocity $V_i=5 \times 10^7$ cm/s and $\alpha_d=2.0$, at different laser wavelengths of 0.21, 0.26, and 0.35 $\mu$m. Their average densities and maximum areal densities are shown in Fig. 7, and closely follow the analytic scaling relations of (19) and (18).

B. In-flight aspect ratio

Another important hydrodynamic parameter determining target performance is the in-flight aspect ratio (IFAR), which is defined as the maximum value of the ratio between the average shell radius and the in-flight thickness. The IFAR determines the Rayleigh-Taylor (RT) instability growth on the outer shell surface during the acceleration phase, and it is commonly used (Ref. 2, p. 3961) to assess the stability properties of ICF implosions. The growth rate of the RT instability for a typical direct-drive DT capsule can be approximated by $\gamma_{RT}=0.94 \sqrt{k g - 2.7 k V_a}$, where $k$ is the mode wave number, $g$ is the acceleration, and $V_a$ is the ablation velocity defined in Sec. II. The RT growth factor is $\exp(\gamma t)$ and $\gamma t = N_f$ is the number of e-foldings. Using the growth rate formula, the number of e-foldings can be written in the following form:

$$N_f = 1.33 \sqrt{\mu(k\Delta_d)\text{IFAR} - 5.4\mu(k\Delta_d)\text{IFAR}(V_a/V_i)},$$

where $\mu=D/R$ is the ratio between the distance $D$ traveled by the shell during the acceleration phase and the target radius $R$.

The aspect ratio IFAR follows the Mach number scaling law as $\text{IFAR} \sim \text{Mach}_d^2$ (Ref. 2, p. 3960), obtained by scaling the shell kinetic energy $MV^2/2$ with the work done by the ablation pressure $\sim P_{\text{ab}} R^3$. A simple manipulation leads to the following scaling relation:

$$\text{IFAR} \sim \frac{V_i^2}{(\alpha_d)^{0.72}} \left(\frac{L}{3 \times 10^7}\right)^{2.12} \left(\frac{\lambda_L}{0.35}\right)^{0.27},$$

as shown in Fig. 8. The shell thickness used in the IFAR is defined as the distance between the inner and outer points where the density is $1/e$ times the maximum value. Such a definition of the IFAR leads to values that are about 20% below the maximum IFAR. Notice that, in flat adiabat implosions, the average adiabat is the same as its inner value; i.e., $\langle \alpha_d \rangle = \alpha_{\text{in}}$. Instead, shaped adiabat targets have approximately $\langle \alpha_d \rangle = 1.6 \alpha_{\text{in}}$, thus leading to lower IFARs and better stability for the same inner surface adiabat. Equation (22) indicates that high in-flight adiabat, low velocity shells have low IFARs; hence, they are less sensitive to the hydrodynamic instability growth during the implosion.

A simple form of the number of RT e-folding (20) can be obtained by approximating the ablation velocity with $V_a \approx 4.3 \times 10^7 \alpha_{\text{out}}^{3/5} \lambda_L^{14/15}$. For shaped adiabat implosions, $\alpha_{\text{out}}$ is the adiabat on the outer shell surface where the laser ablation takes place and the RT instability develops. Since the mode numbers causing shell break-up are those with $k\Delta_d \approx 1$, the number of e-foldings for those modes is obtained by substituting (22) into (20) yielding

![Fig. 6. (Color online) Peak areal density $\rho R$ from simulations (dots) compared to the numerical fit in Eq. (19) (solid line).](image1)

![Fig. 7. (Color online) Density and areal density dependence on laser wavelength. Equations (18) (density, dashed curve) and (19) (areal density, solid curve) are plotted for a 100-kJ implosion with $V_i=5 \times 10^7$ cm/s, $\alpha_d=2$ against laser wavelength $\lambda_L$. The solid triangles (density) and squares (areal density) represent the numerical results for the same implosion with $\lambda_L =0.21, 0.26, and 0.35 \mu$m, respectively.](image2)
expressed as functions of the hot-spot pressure $p_h$ and radius $R_h$. The scaling relations for the hot-spot mass, areal density, central density, and central temperature are, respectively, as follows:\[23\]

\[M_h(t) = \int_0^t p_h(t')^\gamma R_h(t')^3 \gamma p_h(t')^{(\gamma+1)/\gamma} dt',\]

\[\rho R_h(t) = \int_0^{R_h} \rho dr \sim \frac{M_h(t)}{R_h(t)^{3/2}},\]

\[\rho_h(t) \sim \frac{M_h(t)}{R_h(t)^3},\]

\[T_h(t) \sim \frac{p_h(t)}{\rho_h(t)} \sim \frac{p_h(t) R_h(t)}{\rho R_h(t)},\]

where $t=0$ is the starting time of the deceleration phase, and $\nu$ is the Spitzer thermal conductivity. For $\gamma=\frac{7}{5}$, $\nu=\frac{7}{5}$, the index $\beta$ has a constant value of $\beta=\frac{7}{5}$. Notice that the hot-spot mass increases with time due to ablation off the inner-shell surface, even though the hot spot is an insulated system from the energetic perspective.

In order to model the evolution of the hot spot under the action of the cold imploding shell, the latter is approximated by a thin, uniform, incompressible layer of high-density material with mass $M_s$ and initial velocity $V_i$. The implosion shell is considered as a rigid spherical piston compressing and heating the hot spot during the deceleration phase. It converts its kinetic energy into hot-spot internal energy via $pdV$ work. Once a sufficiently large pressure is reached inside the hot spot, thermonuclear ignition is triggered by the locally deposited power from the alpha particles, leading to a fast increase of the hot-spot energy. In typical ICF implosions, the shell is thick, compressible, and nonuniform. Its dynamics is governed by a set of compressible flow relations referred to as the “thick shell model.”\[15\] Nevertheless, even the simple thin shell model used here provides an adequate qualitative description for the shell interaction with the hot spot and it can be used in the derivation of the scaling relations. The thin shell slows down under the pressure of the hot spot, and its dynamics is governed by Newton’s law,

\[M \ddot{R}_h = 4\pi R_h^2 \ddot{p}_h.\]

Combining this equation with the adiabatic relation $p_h R_h^{3\gamma} \approx \text{const}$ yields a single ordinary differential equation describing the hot-spot radius evolution in time

\[\dot{R}_h \frac{d^2 \dot{R}_h}{dt^2} = 1,\]

where $t=0$ corresponds to the starting time of the deceleration phase, and $\dot{t}$ and $\dot{R}_h$ are a dimensionless time and hot spot radius, respectively:

\[\text{FIG. 8. (Color online) In-flight aspect ratio from simulations (dots) compared to the numerical fit in Eq. (22) (solid line).} \]

\[N_e \approx \frac{V_i}{3 \times 10^7} \left[ \frac{7}{\langle \alpha_i \rangle^{0.36} \gamma_{15}^{0.35}} \left( \frac{\lambda_L}{0.05} \right)^{2/15} \right. \]

\[\left. - \frac{0.58}{4^{1/3} \langle \alpha_i \rangle^{0.6} \gamma_{15}^{0.72}} \right] . \]

Equation (23) shows that $N_e$ is proportional to the implosion velocity; hence, high implosion velocities lead to large growth factors for the most dangerous modes. It is important to notice that adiabat shaped targets with $\alpha_{in} > \alpha_{inn}$ (thus, $\langle \alpha_i \rangle > \alpha_{inn}$) have better stability by reducing the RT drive [first term on the right-hand side of (23)] and augmenting the ablative stabilization [second term on the right-hand side of (23)].

V. STAGNATION HOT-SPOT PROPERTIES

The hot spot is a low-density plasma heated by the $pdV$ work of the cold, dense surrounding shell. In the following derivation, to distinguish hot-spot and shell quantities, we use the subscript $h$ when referring to quantities of the central hot spot, and the subscript “s” when referring to those of the surrounding cold shell. The superscript “stg” represents the corresponding quantity at stagnation. The hot spot consists of the ionized DT gas and the plasma ablated off the inner shell surface. By neglecting bremsstrahlung radiation energy losses and alpha particle heating, and assuming that the heat conduction losses are fully recycled into the hot spot by the ablation process,\[23\] the hot spot can be described as an adiabatic system satisfying $p_h V_h^2 = p_h R_h^3 \approx \text{const}$. In this model, the only energy transfer is between the hot-spot internal energy and the shell kinetic energy. This assumption is only valid up to the onset of ignition. At ignition, the alpha particle heating is greater than the energy losses, and the hot spot is no longer adiabatic.

Analyzing the self-similar flow inside the hot spot,\[15,23\] all the relevant hot-spot hydrodynamic parameters can be
\[ \hat{R}_h = \frac{R_h}{R_h(0)}, \quad \hat{V}_i = \frac{V_i}{V_i(0)}, \quad \tau = \frac{4 \pi p_h(0) R_h(0)}{M_i \tau}. \]  

By defining the dimensionless implosion velocity as \( \hat{V}_i = \frac{V_i}{V_i(0)} \), the initial conditions of Eq. (29) can be written as
\[ \hat{R}_h(0) = 1, \quad \hat{V}_i = 0, \quad \hat{V}_d(0) = 1. \]  

Notice that in the initial conditions, the time derivative is with respect to \( \hat{t} \). \( R_h(0) \) is the hot-spot radius, and \( V_i \) is the shell implosion velocity at the beginning of the deceleration phase. Equation (29) can be analytically solved, yielding
\[ \hat{R}_h(\hat{t}) = \sqrt{1 - 2 \hat{V}_i \hat{t}^2 + \hat{t}^2(1 + \hat{V}_i^2)}, \]  

and leading to the following values of the stagnation time and radius:
\[ \hat{r}_{stg} = \frac{\hat{V}_i}{1 + \hat{V}_i^2}, \quad \hat{R}_{stg} = \frac{R_h(0)}{1 + \hat{V}_i^2}. \]  

The hot-spot pressure is derived from the adiabatic condition
\[ p_h(\hat{t}) = \frac{p_h(0)}{[1 - 2 \hat{V}_i \hat{t}^2 + \hat{t}^2(1 + \hat{V}_i^2)]^{5/2}}, \]  

yielding the stagnation value
\[ p_{stg}^h = p_h(0)(1 + \hat{V}_i^2)^{5/2}. \]  

Using Eqs. (24)–(35), it is straightforward to derive scaling relations for the hot-spot variables at stagnation. Since the hot spot is adiabatic, we set \( p_h(t)R_h(t) = \hat{p}_{stg}(\hat{R}_{stg})^3 \) into (24) to find the scaling relation for the hot-spot mass at stagnation
\[ M_{stg}^h \sim \left( \frac{\hat{p}_{stg}(\hat{R}_{stg})^3}{\pi(\hat{p}_{stg})^{5/3}(\hat{R}_{stg})^{5/3}} \right)^{1/(r+1)} \int_0^{\hat{r}_{stg}} p_h(\hat{t})^{3/2} d\hat{t}. \]  

Substituting the hot-spot pressure history during the deceleration phase (34), leads to the following simple form of the time integral in (36):
\[ \int_0^{\hat{r}_{stg}} p_h(\hat{t})^{3/2} d\hat{t} \sim p_h(0)^{5/2} \hat{V}_i^2, \]  

where only the highest order term in \( \hat{V}_i \) is retained for large implosion velocities (\( \hat{V}_i \gg 1 \)).

The in-flight values of the hydrodynamic variables are related to the stagnation values through the thin shell model. Equations (33) and (35) for the hot-spot radius and pressure at the beginning of the deceleration phase, can be written in the following form:
\[ R_h(0) \sim R_{stg}^h \hat{V}_i, \]  

\[ p_h(0) \sim p_{stg}^h \hat{V}_i^5. \]  

Substituting (38) and (39) into (30), and rewriting the quantity \( \tau p_h(0)^{5/2} \) as
\[ \tau p_h(0)^{5/2} \sim M_i^{1/2}(p_{stg}^h)^{3/10}(R_{stg}^h)^{-1/2} \hat{V}_i^2 \]  

leads to the following form of the hot-spot mass:
\[ M_{stg}^h \sim M_i^{1/7}(p_{stg}^h)^{4/7}(R_{stg}^h)^{16/7}. \]  

It is important to distinguish the two masses: \( M_{stg}^h \) and \( M_s \). \( M_{stg}^h \) is the hot-spot mass at stagnation, consisting mostly of the ablated material from the inner shell surface. \( M_s \) is the cold shell mass at the beginning of the deceleration phase, and it scales with the shell kinetic energy \( E_k = M_i \hat{V}_i^2/2 \). Substituting (41) into (25) and (27) leads to the following relations for the hot-spot areal density and temperature in terms of its stagnation properties
\[ \rho R_{stg}^h \sim M_i^{1/7}(p_{stg}^h)^{1/7}(R_{stg}^h)^{-27/7}, \]  

\[ T_{stg}^h \sim M_i^{1/7}(p_{stg}^h)^{1/7}(R_{stg}^h)^{-5/7}. \]  

For an isobaric assembly, the stagnation pressure is uniform throughout the hot spot and the shell (\( p_{stg}^h = \rho_s \)), so that the hot-spot stagnation pressure can be related to the shell stagnation properties through the shell adiabat: \( p_{stg}^h = \rho_s \). Thus, substituting (11) and (12), yields the stagnation pressure scaling law,
\[ p_{stg}^h \sim \Psi(A_i)^{5/3} \alpha_{il}^{-6/5} V_i^{4/3} L_i^{15/13} T_L^{-2/15}. \]  

Using the shell stagnation aspect ratio definition, the hot-spot radius is \( R_{stg}^h = A_i \Delta s \). Setting \( M_i \sim E_k V_i^{-2} \), the scaling laws for the stagnation hot-spot properties are obtained upon substitution of (10)–(12) and (44) into (42) and (43),
\[ \rho R_{stg}^h \sim \Omega(A_i) \alpha_{il}^{-4/7} V_i^{34/21} L_i^{16/3} T_L^{-4/63}, \]  

\[ T_{stg}^h \sim \pi(A_i) \alpha_{il}^{-35/3} V_i^{22/21} L_i^{8/315} T_L^{-8/315}, \]  

where the functions \( \Omega(x) \) and \( \Pi(x) \) are defined as \( \Omega(x) = x^{57/30} / (1 + x)^{27/7} \), and \( \pi(x) = x^{-7/30} / (1 + x)^{5/7} \). These two functions can be approximated as \( \Omega(A_i) \sim 1/A_i^{0.44} \), and \( \Pi(A_i) \sim \text{const.} \) for \( A_i \) within the range of \( 1 < A_i < 4 \) for typical ICF implosions. Thus, by using (4) and (15), the scaling laws in (44)–(46) can be rewritten in their final form,
\[ p_{stg}^h \sim \alpha_{il}^{-1.40} V_i^{3.02}, \]  

\[ \rho R_{stg}^h \sim \alpha_{il}^{-0.49} V_i^{1.38} L_i^{5/21}, \]  

\[ T_{stg}^h \sim \alpha_{il}^{-0.23} V_i^{1.2} E_k^{2/21}. \]  

The laser intensity and wavelength dependence are disregarded due to the small power indices. Notice that the hot-spot temperature depends mostly on the implosion velocity. The faster the target is driven, the higher the temperature can rise in the hot spot. This occurs because the compression work is supplied at a rate proportional to the implosion velocity. For large implosion velocities, the work rate overcomes the thermal conduction losses; hence, the temperature increases.

The scaling relations (47)–(49) are compared with the numerical results from the simulated implosion database. The database is used to derive scaling relations for the stag-
nation pressure, stagnation hot-spot temperature, the hot-spot $\rho R$, the shell $\rho R$, and the stagnation shell density. The position of the shell inner and outer surfaces is set by the local minimum of the density gradient scale length $L = [(1/\rho) \times (\text{d}\rho/\text{d}r)]^{-1}$ across the fuel. From the center outward, the first minimum encountered represents the inner shell surface that also corresponds to the hot-spot radius $R_h$. The second minimum is due to the outward going return shock, representing the outer shell surface. The distance between these two minima is defined as shell thickness $\Delta_r$. The hot-spot $\rho r$ is the maximum areal density from the center to $R_h$ achieved during the implosion; i.e., $\rho r = \max \int_0^{R_h} \rho \text{d}r$. The hot-spot temperature $T_h$ and pressure $p_h$ are the maximum temperature and pressure averaged over the hot-spot volume, $\langle T_{h,p_h} \rangle = \int_0^{R_h} (T_{h,p_h}) \text{d}r^3/R_h(t)^3$. The numerical fit derived from the simulated implosion database is shown in Figs. 9–11, and the scaling relations are given below:

$$\langle p \rangle_{\text{hot-spot}}^\text{fit} = \frac{345}{\alpha_{\text{inn}}^{0.96}} \left( \frac{V_i}{3 \times 10^7} \right)^{1.85},$$

$$\rho R_{\text{hot-spot}}^\text{fit} = \frac{0.31}{\alpha_{\text{inn}}^{0.55}} \left( \frac{V_i}{3 \times 10^7} \right)^{0.62} \left( \frac{E_L}{100} \right)^{0.27},$$

$$\langle T \rangle_{\text{hot-spot}}^\text{fit} = \frac{3.0}{\alpha_{\text{inn}}^{0.15}} \left( \frac{V_i}{3 \times 10^7} \right)^{1.25} \left( \frac{E_L}{100} \right)^{0.07},$$

where the pressure $p$ is in gigabar, the areal density $\rho R$ in g/cm$^2$, and the temperature $T$ in keV. Notice that Eqs. (50)–(52) are in good qualitative and sometimes quantitative agreement with the analytic scaling laws of (47)–(49). It is important to emphasize that the fusion burn is not included in the simulations. This leads to hot-spot temperatures lower than the one in ignition targets due to the lack of the alpha particle heating. Thus, (50) and (52) represents a lower bound of the estimated hot-spot temperature and pressure that can be achieved in high performance implosions. While Eq. (52) represents the volume averaged hot-spot temperature, the central temperature $T_0$ follows a similar scaling law as $T_0 \sim V_i^{1.3}/\alpha^{0.06}$.

VI. SUMMARY AND DISCUSSION

As shown in Secs. III–V, all the analytic scaling laws are in qualitative agreement with the numerical fits. The maximum $\rho R$, the hot-spot temperature and the shell density, are also in good quantitative agreement. The hot-spot $\rho R$ and the maximum pressure show significant discrepancies with the numerical fits, particularly in their velocity scaling. An analytic scaling of the pressure and shell density was also de-
TABLE I. Summary of the scaling relations derived in this paper. All the variables are calculated in the absence of alpha-particle self-heating.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Scaling relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrodynamic efficiency</td>
<td>( \eta = \frac{0.051}{\alpha_{\text{lin}}^{0.25}} \left( \frac{V}{3 \times 10^7} \right)^{0.75} \left( \frac{\rho}{100} \right)^{0.25} \left( \frac{\lambda}{\mu m} \right)^{0.5} )</td>
</tr>
<tr>
<td>Thermonuclear gain</td>
<td>( G = \frac{365}{\alpha_{\text{lin}}^{0.25}} \left( \frac{V}{3 \times 10^7} \right)^{1.25} \left( \frac{\rho}{7 \times \rho R} \right)^{0.35} \left( \frac{\lambda}{\mu m} \right)^{0.5} )</td>
</tr>
<tr>
<td>Shell areal density [g/cm²]</td>
<td>( (\rho R)<em>{\text{max}} = \frac{1.2}{\alpha</em>{\text{lin}}^{0.04}} \left( \frac{V}{3 \times 10^7} \right)^{0.33} \left( \frac{\rho}{100} \right)^{0.35} \left( \frac{\lambda}{\mu m} \right)^{0.25} \left( \frac{V}{3 \times 10^7} \right)^{0.06} )</td>
</tr>
<tr>
<td>Shell density [g/cm³]</td>
<td>( (\rho)<em>{\text{sh}} = \frac{425}{\alpha</em>{\text{lin}}^{0.04}} \left( \frac{V}{3 \times 10^7} \right)^{0.13} \left( \frac{\rho}{100} \right)^{0.35} )</td>
</tr>
<tr>
<td>Shell IFAR</td>
<td>( \text{IFAR} = \frac{400}{\left( \alpha_{\text{lin}} \right)^{0.27}} \left( \frac{V}{3 \times 10^7} \right)^{2.12} \left( \frac{\rho}{100} \right)^{0.27} \left( \frac{\lambda}{\mu m} \right) )</td>
</tr>
<tr>
<td>Hot-spot areal density [g/cm²]</td>
<td>( \rho R_h = \frac{0.31}{\alpha_{\text{lin}}^{0.25}} \left( \frac{V}{3 \times 10^7} \right)^{0.62} \left( \frac{\rho}{100} \right)^{0.27} \left( \frac{\lambda}{\mu m} \right) )</td>
</tr>
<tr>
<td>Hot-spot temperature [keV]</td>
<td>( \langle T_h \rangle = \frac{3.00}{\alpha_{\text{lin}}^{0.13}} \left( \frac{V}{3 \times 10^7} \right)^{1.25} \left( \frac{E_i}{100} \right)^{0.07} )</td>
</tr>
<tr>
<td>Hot-spot pressure [Gbar]</td>
<td>( \langle p_h \rangle = \frac{345}{\alpha_{\text{lin}}^{0.90}} \left( \frac{V}{3 \times 10^7} \right)^{1.85} )</td>
</tr>
<tr>
<td>Stagnation aspect ratio</td>
<td>( A_s = \frac{1.48}{\alpha_{\text{lin}}^{0.19}} \left( \frac{V}{3 \times 10^7} \right)^{0.96} )</td>
</tr>
</tbody>
</table>

rived in Refs. 16 and 17. For instance, the scaling of Basko and Meyer-ter-vehn16 yields \( \rho \sim \rho_0 \text{Mach}_i^4 \) and \( \rho \sim \rho_0 \text{Mach}_i^5 \), where Mach is the in-flight Mach number, with a much stronger velocity dependence than predicted by our simulations. The difference between Ref. 16 and our analytic relations is due to the stagnation aspect ratio \( A_s \) scaling used in the derivation. In Ref. 16, the stagnation aspect ratio scales as \( A_s \sim \text{Mach}_i \), while in our analysis it scales approximately as \( A_s \sim \text{Mach}_i \). Such a scaling of the stagnation aspect ratio is suggested by the numerical fit shown in Fig. 4. Other discrepancies between the analytic and numerical scalings may arise because of the simplified ideal gas equation of state used in the analytic derivation. Instead, all the numerical scalings are derived from implosion simulations using SESAME equation of state tables.

In this section, we will make use of the more accurate numerical fits summarized in Table I. Equations (Table I) relating the stagnation to the in-flight values of the hydrodynamic variables are useful to guide the design of conventional and fast ignition direct-drive ICF targets and laser pulses. Starting from the gain relation (5) and the areal density scaling (19), it follows that, for a fixed laser energy \( E_L \), high gains can be achieved through low implosion velocities (i.e., massive targets) and low inner-surface in-flight adiabats \( \alpha_{\text{lin}} \). It is important, however, to emphasize that both conventional and fast ignition implosions require a minimum implosion velocity for ignition at a fixed adiabat. In conventional ICF, the minimum implosion velocity is set by the 1-D marginal ignition condition requiring that the minimum shell kinetic energy for achieving a gain of unity14–18 in one dimension scales as \( E_k \sim \alpha_{\text{lin}}^{0.9} V_i^{0.5} P_L^{0.77} \), where \( P_L \) is the laser-applied pressure at the end of the acceleration phase. Using the hydrodynamic efficiency in (4), the laser energy required for marginal direct-drive ICF ignition can be written as

\[
E_L^{\text{min}} = \frac{0.25}{\alpha_{\text{lin}}^{0.9}} \left( \frac{3 \times 10^7}{V_i} \right)^{1.9} \left( \frac{\lambda}{0.35} \right)^{0.5} \left( \frac{100}{P_L} \right)^{0.77},
\]

where \( E_L^{\text{min}} \) is in megajoules, and the peak ablation pressure \( P_L \) (in megabars) is related to the peak laser intensity \( \left( P_L \sim I_L^{1.2} \right) \) according to the planar direct-drive scaling. For a given adiabat, laser wavelength, laser intensity, and laser energy, Eq. (53) can also be used to determine the minimum implosion velocity required for ignition \( V_i^{\text{min}} \). Standard ICF ignition targets require an implosion velocity at least \(~10\%–20\%\) higher than \( V_i^{\text{min}} \) in order to overcome the deterioration of the hot-spot ignition conditions due to the deceleration phase Rayleigh-Taylor instability. A higher implosion velocity requires a greater driver energy leading to a shell kinetic energy \(~20\%–50\%\) above the 1-D marginal ignition value (this energy surplus is usually referred to as “margin”18). If the laser energy is fixed, the margin can be
produced by lowering the shell mass, hence increasing the implosion velocity and decreasing the marginal ignition energy. As indicated by the gain formula (5), any increase in the implosion velocity inevitably leads to a decrease in the energy gain (assuming that ignition takes place) despite the fact that the burn fraction does not change since the areal density is approximately independent of the implosion velocity. Furthermore, any enhancement of the implosion velocity leads to an increase of the IFAR (IFAR \( \sim V_i^2 \)) leading to the amplification of the most dangerous RT modes during the acceleration phase. It is important to notice that substituting the IFAR scaling (21) into Eq. (53), yields that the minimum energy required for ignition scales approximately as \( E_L^{\text{min}} \sim 1/\text{IFAR}^3 \) and the condition on the minimum implosion velocity is in reality a condition on the minimum IFAR required for ignition. Since the stability properties depend mostly on the IFAR, the in-flight aspect ratio is a key implosion parameter that needs to be carefully chosen within a narrow range in order to minimize the ignition energy while reducing the growth of the RT instability. Typical values of the IFAR in conventional ICF target designs range from 35 to 45. Once the IFAR is set, the implosion velocity follows the scaling in (21): \( V_i \sim \text{IFAR}^{0.5}(\alpha_{\text{ad}})^{0.3} \). Since the gain increases for lower implosion velocities, and the areal density increases for lower adiabats, it is convenient to operate at the lowest possible adiabat and therefore minimum implosion velocity. Since the hot-spot temperature decreases with the implosion velocity (52), it is important to avoid implosion velocities below \( \sim 2 \times 10^7 \text{ cm/s} \), leading to relatively cold hot spots where the radiation losses overcome the alpha heating thus preventing ignition and energy gains. Typical conventional ICF targets are designed with an inner surface adiabat of \( \alpha_{\text{inn}} \sim 1-3 \) and implosion velocities of \((3-4) \times 10^7 \text{ cm/s} \) driven by lasers with energies exceeding 1 MJ. One-dimensional simulations of the implosion predict that the energy gain for a 1–1.5 MJ UV laser varies between 10 and 20 (Ref. 24) for indirect drive and between 40 and 50 (Ref. 25) for direct drive. Using the UV direct-drive National Ignition Facility (NIF) point design parameters of \( \alpha_{\text{inn}} \approx 2.7 \), \( V_i = 4.3 \times 10^7 \text{ cm/s} \), \( I_{15} = 1 \), and \( E_L = 1.5 \text{ MJ} \) into Eqs. (19) and (5), one finds a maximum areal density of 1.77 g/cm\(^2\) and energy gain of 47, in good agreement with the results of Ref. 25.

In fast ignition, the minimum implosion velocity is set by the adiabat and the density required for fast ignition. Since the density scales linearly with the velocity (18), the (monoenergetic) electron beam energy required for ignition scales as \( E_{\text{beam}} \sim 1/\rho^{1.85} \sim (\alpha_{\text{inn}}/V_i)^{1.85} \). As in conventional ICF, optimized fast ignition implosions require low values of the inner-surface in-flight adiabat. As long as the ratio \( V_i/\alpha_{\text{inn}} \sim \rho \) is sufficiently large to achieve the densities required for fast ignition, the implosion velocity can be minimized by driving the shell on the lowest possible adiabat. However, very low adiabat implosions require long pulse lengths and careful pulse shaping. The long pulse length is due to the slow velocity of the low adiabat shocks and the careful shaping is required to prevent spurious shocks from changing the desired adiabat. Furthermore, the ratio between the peak power and the power in the foot of the laser pulse (i.e., the power contrast ratio) increases as the adiabat decreases, thus leading to difficult technical issues in calibrating the pulse shape. These constraints on the pulse shape are alleviated by using the relaxation laser pulse technique. As suggested in Ref. 4, reasonable minimum values of the inner-surface adiabat and implosion velocity are \( \alpha_{\text{inn}} \approx 0.7 \) and \( V_i = 1.7 \times 10^7 \text{ cm/s} \). An adiabat below unity implies that at shock break-out, the inner portion of the shell is not fully ionized. According to the scaling relation (18), the average density corresponding to such values of the adiabat and velocity is about 350 g/cc, which is typically considered within the optimum range for fast ignition.\(^{5,12} \) In order to achieve the high areal densities of \( \rho R \approx 3 \text{ g/cm}^2 \) required for IFE, the laser driver energy can be derived from the scaling relation (18), leading to \( E_L \approx 870 \text{ kJ} \). The corresponding energy gain can be derived from the gain formula (5) after including a reduction factor,\(^{12} \) for the areal density available for the burn \( \rho R_{\text{burn}} \approx 0.7 \rho R_{\text{max}} \), leading to \( G \approx 170 \) for \( E_L = 870 \text{ kJ} \), \( V_i = 1.7 \times 10^7 \text{ cm/s} \), and \( \alpha_{\text{inn}} \approx 0.7 \). Using low-velocity implosions of massive shells for fast ignition fuel assembly should also improve the performance of cone-in-shell targets where a gold cone is inserted into the shell to keep a plasma-free path for the fast ignitor pulse.\(^{26,27} \) Recent experiments and simulations of cone-in-shell target implosions\(^{28} \) have shown that the integrity of the cone tip is compromised by the large hydrodynamic pressures and that a low-density plasma region develops between the cone tip and the dense core thus complicating the fast electron transport. Since the stagnation pressure scales as \( p \sim V_i^{1.9} \), the fuel assemblies from low-velocity implosions can improve the cone target performances since the resulting dense core has relatively low pressure (due to the low velocity), thus reducing the hydrodynamic forces on the cone tip. Furthermore, since low velocities are obtained by imploding shells with large masses, the resulting core size is large thus reducing the distance between the tip and the dense core edge.

Finally, it is important to point out that hydrodynamic instabilities can alter some of the scaling relations. While the areal densities and pressure are expected to be rather insensitive to shell distortions, the hot-spot temperature significantly decreases in a nonuniform compression due to the augmented heat-transfer area. Since the growth of the Rayleigh-Taylor instability depends on the in-flight aspect ratio (IFAR), we speculate that the 3-D temperature scaling will likely exhibit an adverse relationship with the IFAR leading to \( T_{3-D} \sim T_{1-D}(\text{IFAR}) \), where \( f(\text{IFAR}) \) decreases with the IFAR.

In summary, the hydrodynamic relations (Table I) for hydro-efficiency, thermonuclear gain, in-flight aspect ratio, shell density, areal density, hot-spot areal density, pressure, and temperature derived in this paper, can be used to optimize the design of conventional and fast ignition ICF targets. These relations are derived both analytically and numerically using a simulated implosion database built with the one-dimensional hydrodynamic code LILAC.
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