Steady-state planar ablatif flow

W. M. Manheimer and D. G. Colcombant
Plasma Theory Branch, Plasma Physics Division, Naval Research Laboratory, Washington, D.C. 20375

J. H. Gardner
Laboratory for Computational Physics, Naval Research Laboratory, Washington, D.C. 20375

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Steady-state planar ablatif flow in a laser produced plasma is studied. The calculations relate all steady-state fluid quantities to only three parameters, the material, absorbed irradiance, and laser wavelength. The fluid is divided into three regions; the subcritical expanding plasma, the steady-state ablation front, and the accelerated slab. Boundary conditions at the interfaces of these regions are given. If the absorbed irradiance is nonuniform, the nonuniformity in ablation pressure is calculated. Results are compared with experiment and fluid simulation for both uniform and nonuniform illumination.

I. INTRODUCTION

Recently, there have been a series of experiments on laser driven ablatif acceleration of thin foil targets at the Naval Research Laboratory and elsewhere. These experiments demonstrated acceleration of the foil up to speeds in excess of $10^7$ cm/sec. The authors also compared the experimental results with a simple rocket model. The laser irradiances in these experiments are sufficiently low so that the plasma is most likely described by classical transport.

In this paper we discuss the theory of ablatif acceleration. Since this problem can be solved in either one or two dimensions by fluid simulation, our principal object is not to find detailed solutions which can be compared precisely with experiment. Rather, it is to shed light on the physics of ablatif acceleration and to derive simple scaling laws. We do derive simple scaling laws for ablation pressure, blow-off velocity, and separation between critical and ablation surfaces. These scaling laws depend only on the material, laser wavelength, and absorbed laser irradiance. The key to deriving these simple scaling laws is not really in solving the steady state fluid equations, but rather in selecting which of many possible solutions describe the ablatif acceleration.

Another crucial question for ablatif-driven laser fusion is what degree of nonuniformity of illumination can be tolerated. Initial experiments in this area have also been done at the Naval Research Laboratory. This paper also addresses the issue of ablatif undisturbed fluid.

The earliest theories of uniform laser driven acceleration assumed that the critical surface behaved like a Chapman–Jouguet deflagration point. The dense, heated plasma then acts as a piston and drives a shock into the cold undisturbed fluid. The laser energy is ultimately coupled to the target by this shock wave. Except for Refs. 16 and 20, these theories neglect thermal conduction. They show that the ablation pressure $P_a$ scales as $I^{1/3} \lambda^{2/3}$ and the blow-off velocity scales as $I^{1/3} \lambda^{1/3}$, where $I$ is the absorbed irradiance and $\lambda$ is the laser wavelength. They assume, as we do, that the laser light is absorbed only at the critical density. These scaling laws are similar to the ones we derive. This is interesting since thermal conduction is neglected. In ablatif flow, thermal conduction is the principal inward energy transport mechanism.

More recent theories include the effects of thermal conduction. These theories were done in either planar or spherical geometry. Since the thermal conduction is in fact very large in the blow-off plasma, we also include thermal conduction. In these newer theories (except Ref. 24), the scaling laws are either not explicitly given, or else they tend to be very complicated. For instance, the scaling law for ablation pressure given in Ref. 23 is

$$P_a = I^{1/3} \lambda^{2/3} \left( \frac{M_a^2 - 24 \pi R_a^2 - 2 \pi^2}{\gamma M_a^2 + 7 \pi^2} \right)^{1/2},$$

where $M_a$ is the Mach number at which the thermal flux is inhibited, $R_a$ is the absorbed laser irradiance at the ablation surface, and $R_a$ is the ablation surface radius. On the other hand, Ref. 22 finds that steady-state solutions can only be found if their parameter $M$ (proportional to critical-ablation surface separation in our theory) is between 2/3 and 16/3. Also, they find that steady-state solutions exist at given critical to sonic density ratio, only for a particular value of absorbed irradiance, in other words, the absorbed irradiance cannot be independently specified.

Our solutions are more like those calculated for planar geometry. However, the solutions presented here more clearly connect the flow to the laser parameters and show explicitly how the slab accelerates. In fact, we find that the acceleration is simply $P_a / M$, where $M$ is the mass of the slab which has not yet ablated away. Also, the flows calculated in Ref. 21 require several more parameters to be specified. For instance, solutions presented in Ref. 21 specify the temperature at the critical surface and acceleration whereas we solve for these. We need only specify the material, laser wavelength, and absorbed irradiance. The calculations in Ref. 21 are also more difficult to apply to an experiment because fluid quantities are normalized to the values they have at the maximum density which is not known a priori.

We find that the fluid can most easily be described by breaking it into three regions. First, there is the subcritical plasma which expands into a vacuum. Since it expands into a vacuum, it cannot be in a steady state; rather, the expansion will be some sort of rarefaction wave. Since the thermal conduction is very high, an isothermal rarefaction is most reasonable.
Second, there is the ablation front between the critical surface and the accelerated slab. We assume that the flow is in steady state there. The steady-state ablation front can match smoothly to the nonsteady rarefaction if the isothermal Mach number is unity at the critical surface. This feature also corresponds to what is found in particle simulations and the theory of laser light interaction with the critical surface. The analytic solution for fluid quantities in the ablation front tends to zero temperature a finite distance away from the critical surface. This point then marks the transition from the ablation front to accelerated slab. These two regions are described in Sec. II.

Third, there is the accelerated slab itself. In Sec. III we review Kidder's theory for an accelerated degenerate Fermi-Dirac gas. The structure near the transition point is almost impossible to calculate analytically since the material makes a transition there from degenerate Fermi gas to fully ionized plasma. In this work, we treat the transition as a contact discontinuity across which mass, momentum, and energy flux are conserved, but do not concern ourselves with its detailed structure. Also, we determine under what conditions the acceleration is negligible in the calculation of the properties of the ablation front.

In Sec. IV we examine the problem of nonuniform illumination. First, it is assumed that the one-dimensional flow pattern is stable so two-dimensional steady-state treatment is meaningful. If we assume that all quantities have a small transverse perturbation, the steady-state fluid equations can be linearized in the perturbed quantities. These linearized equations turn out to be a set of equations for an isothermal sound wave coupled to an equation for a thermal conduction wave. Approximate solutions to these equations are found which directly relate the nonuniformity of the absorbed irradiance to the nonuniformity of the ablation pressure at the slab. Since the velocity of the slab is \( P_a \) (the ablation pressure) times the pulse time divided by the mass per unit area, we can simply derive velocity nonuniformity in terms of illumination nonuniformity.

In Sec. V we derive simple scaling laws for a laser produced plasma and compare the theory with Naval Research Laboratory experiments for both uniform and nonuniform illumination. Also, we show that the one-dimensional solutions calculated are very close to what is found by a one-dimensional numerical solution. As we shall see, the theory compares reasonably well with experiment.

II. STEADY-STATE ABLATION IN ONE DIMENSION

In this section, we examine the steady-state flow which results when an intense laser beam illuminates a slab. The configuration is shown in Fig. 1 with the laser on the right and the flow in the positive \( x \) direction. There are three distinct regions in the flow: Farthest to the right is the subcritical density plasma. Since this plasma expands into a vacuum, it cannot be steady state; rather, it will be characterized by some sort of rarefaction wave. Farthest to the left is the accelerating slab which we will discuss in the next section. In between is the region of steady-state ablative flow. As we shall see, and as has been pointed out by others, the solution of the steady-state fluid equations are quite straight-forward. The main problem is to connect these solutions properly from one region to the next, so that one eliminates as many constants of integration as possible. We find that the steady-state ablative flow is characterized completely by the material, laser wavelength, and absorbed laser irradiance.

In the reference frame in which the ablation front is at rest, the steady state conservation equations for mass and momentum are then

\[
\frac{d}{dx} \rho v = 0, \tag{1a}
\]

\[
\rho v = \rho_c \nu_c, \tag{1b}
\]

and

\[
\frac{d}{dx} (\rho v^2 + \rho T) = 0, \tag{2a}
\]

\[
\rho v^2 + \rho T = \rho_c v_c^2 + \rho_c T_c, \tag{2b}
\]

where \( \rho, v, \) and \( T \) are the mass density, velocity, and temperature. For convenience, the temperature has units of velocity squared, so that \( T^{1/2} \) is the isothermal sound speed. Also, we have neglected gravity (acceleration) in the ablation front. We will give later an \emph{a posteriori} justification for this approximation. A subscript \( c \) denotes the value of a quantity at the critical density. Also, we have neglected the ponderomotive force and the inertial force due to the accelerated reference frame. The former approximation is valid if the laser light energy density is small compared with the thermal energy density at the critical surface. We will discuss the validity of the latter approximation in the next section. The critical density \( \rho_c \) is determined by the material and laser frequency, (assuming that the ion is fully stripped) but \( \nu_c \) and \( T_c \) are as yet undetermined. Equations (1) and (2a) or (1) and (2b) can easily be solved to give:

\[
\frac{dv}{dx} = - \frac{1}{v(1 - T/v^2)} \frac{dT}{dx}, \tag{3a}
\]

\[
\mathcal{M} = \left( \nu_c + T_c/\nu_c \right)/T_c^{1/2} \pm \left( \nu_c + T_c/\nu_c \right)^2 T_c^{-1} - 4 \right)^{1/2}, \tag{3b}
\]

where \( \mathcal{M} \) is the isothermal Mach number \( \mathcal{M} = v/\sqrt{T} \). Equation (3a) shows that the flow becomes singular at the sonic point \( \mathcal{M} = 1 \). Thus, the sonic transition cannot be in the region of steady-state ablation.

If we assume that the thermal conduction is high, the expansion of the underdense plasma should be approximate-
ly described as an isothermal, rather than adiabatic, rarefaction wave. This (time dependent) solution is given by

$$\rho = \rho_c \exp\left(-x/T\sqrt{\rho_c} t\right),$$  
(4a)

$$v = T\sqrt{\rho_c} + x/t,$$  
(4b)

where $x = 0$ corresponds to the critical density at all times. Clearly, this isothermal rarefaction wave can connect smoothly to the steady-state ablative flow only if

$$v_c = T\sqrt{\rho_c}.$$  
(5)

Thus, as additional boundary condition for the ablative flow region, we assume that the Mach number is equal to unity at the critical surface. This technique of joining a steady-state solution in the ablation front to a time-dependent rarefaction wave in the underdense plasma was also utilized in Ref. 24. To further examine this hypothesis, we have done fluid simulations (see Sec. V) which confirm that $\mathcal{M} \approx 1$ at the critical surface. Both particle simulations and theory also show that accounting for the deposition of laser momentum at the critical surface removes the singularity and requires $\mathcal{M} \approx 1$ at the critical surface.

Lee et al.\textsuperscript{25} find that there is a density jump at the critical surface and the flow velocity makes a transition from subsonic on the high-density side to supersonic on the low density side. The density and velocity jump both approach zero as the electron oscillating velocity divided by electron thermal speed approaches zero. Actually there will be a small density jump at the critical surface, but we do not consider it here. In the Naval Research Laboratory ablation experiments, the laser irradiance is small enough so that the ponderomotive force is unimportant. Thus, Eq. (5) determines the velocity at the critical surface in terms of the temperature. The temperature will be determined from the energy equation.

Before turning to the energy equation, let us look at the expression for Mach number, Eq. (3b). In the ablative flow region, the flow is subsonic. Also, we expect that near the solid, the temperature will be small. Taking the subsonic solution of Eq. (3b) for $\mathcal{M} \approx 1$ in the limit of small $T$, we find

$$\mathcal{M} \approx T^{1/2}/2T\sqrt{\rho_c},$$  
(6)

where we have used Eq. (5). Thus as the temperature decreases, so does the Mach number.

We now turn to the steady-state energy equation which we write as

$$\frac{d}{dx}\left(\frac{5}{2} \rho v T - KT^{5/2} \frac{dT}{dx}\right) = I_0(x),$$  
(7a)

where $I$ is the absorbed laser energy flux. Integrating in $x$ for $x < 0$,

$$\frac{5}{2} \rho_0 v_0 T - KT^{5/2} \frac{dT}{dx} = S,$$  
(7b)

where $S$ is the total constant energy flux. We have assumed that the thermal conduction is proportional to $T^{5/2}$ as is appropriate for an ionized plasma. In Eq. (7) we have neglected the kinetic energy flux $1/2m v^2$ compared with the enthalpy flux $5/2\rho_0 v_0 T$. At the critical density, this is a $20\%$ error. However, as one approaches the solid, and $\mathcal{M}^2 = u^2/T$ decreases, this approximation gets better and better. Neglecting this term allows us to write a very simple analytic solution to the energy equation.

The next question is what the constant $S$ in Eq. (7b) is. The individual terms on the left-hand side of Eq. (7b) roughly have magnitude $I$, the laser irradiance. There are three possible physical effects which contribute to $S$. First, it describes the heating (or preheat) of the slab. This contribution to $S$ from the preheat is negative since in the configuration of Fig. 1 power travels from right to left to reach the slab.

Since accelerating targets with no preheat are of most interest to laser fusion, we do not consider preheat. Second, there is the negative contribution to $S$ from power deposited at the ablation surface needed to vaporize and ionize the slab. Since this energy is very small compared with flow and thermal energy, we also neglect it. Finally, there is the power needed to accelerate the slab. This contribution to $S$ is positive since in the reference frame of the ablation front the slab has positive velocity but negative acceleration. That is the slab loses energy. However the slab velocity is very small, being the critical velocity times the critical density divided by the solid density. Therefore, we will also neglect the power coming out of the accelerated slab, so that $S = 0$ and Eq. (7b) becomes

$$\frac{5}{2} \rho v T - KT^{5/2} \frac{dT}{dx} = 0.$$  
(7c)

Since $\rho v$ is constant, Eq. (7c) has a simple analytic solution

$$T = \left(T\sqrt{\rho_c} + \frac{25}{4} \frac{\rho v}{K} x\right)^{2/5},$$  
(8)

where $x = 0$ is assumed to be the critical surface. Notice that the equation is valid only for

$$x > -\frac{4}{25} \frac{KT\sqrt{\rho_c}}{\rho v},$$

at which point it is singular in that $T \rightarrow 0$, but $\rho \rightarrow \infty$. The solutions of $T$ vs $x$ (analytical and numerical including the kinetic energy flux) are both shown in Fig. 2. Clearly, the former is a very good approximation to the latter and we will use Eq. (8) for $T$ whenever an analytic expression is needed. The problem now is to determine $T_c$ and to determine the meaning of the singular behavior as $T \rightarrow 0$.

To find $T_c$ we must examine the behavior of the flow near the critical density. Since we neglect the ponderomotive force (i.e., laser momentum deposition), the density and ve-
locity are continuous across the critical density. Since the thermal conduction is large, the temperature is also continuous across the critical density. The only discontinuous quantity then is the temperature gradient, and the discontinuity in it reflects the laser energy deposition at the critical density. Specifically,

$$KT_e^2 \left( \frac{dT}{dx} \bigg|_{x=0} - \frac{dT}{dx} \bigg|_{x=0^+} \right) = I,$$

(9)

where $I$ is the absorbed laser irradiance. The question now is how much of this absorbed irradiance is conducted inward and how much is conducted outward.

The outward flux can be determined from the rarefaction wave solution, Eq. (4). The total flow energy flux $\rho v^2 + \frac{1}{2} \rho v T$ through the critical surface is $3 \rho_c V_c T_c$, where we have assumed $\mathcal{M} = 1$. However, by integrating $\frac{1}{2} \rho v^2 + \frac{1}{2} \rho T$ from $x = 0$ to $\infty$ and taking the time derivative, we find that the energy flux through $x = 0$ needed to drive the rarefaction is $4 \rho_c V_c T_c$. Thus, an additional energy flux $\rho_c V_c T_e$ must be supplied by outward thermal conduction so that

$$-KT_e^2 \left( \frac{dT}{dx} \bigg|_{x=0} \right) = \rho_c V_c T_e.$$ 

(10)

The small temperature gradient in the underdense plasma slightly violates the isothermal assumption. However, for the temperatures we consider, this deviation proves to be extremely small. Hence, the temperature in the underdense plasma is not exactly constant. Our model for the rarefaction wave is valid in the limit $K \to \infty$ so that the relative temperature drop across this rarefaction wave is small. The inward flux can be obtained immediately from Eq. (7) and gives

$$KT_e^2 \left( \frac{dT}{dx} \bigg|_{x=0^0} \right) = \frac{5}{2} \rho_c v_c T_c + \frac{1}{2} \rho_c v_c^3.$$ 

(11)

Thus, making use of $\mathcal{M} = 1$ the conditions at the critical surface are simply given by

$$KT_e^2 \left( \frac{dT}{dx} \bigg|_{x=0^0} \right) = 3 \rho_c T_e^{3/2} = \frac{3}{4} I.$$ 

(12)

The steady-state ablative flow pattern is now determined by only three quantities, the material, the laser frequency, and the absorbed irradiance.

The singularity at $T = 0$ marks the transition from ablation front to accelerated slab. If the slab is Fermi degenerate, the temperature actually does go to zero. Actually, various nonideal effects will surely smooth the temperature profile from ablation front to slab. The mass flux is continuous from ablation front to slab, and the total energy flux is nearly zero in both regions. The ablation pressure at the interface goes into accelerating the slab, as we shall see in the next section.

At this point, it is worth summarizing the scaling laws for ablation pressure $P_a$ and blow-off velocity at the critical surface $v_c$. Equations (2b) and (12) show

$$P_a = \rho_c v_c^2 + \rho_c T_e \sim I^{2/3} \lambda^{-2/3}$$

(13a)

while the Mach = 1 condition gives the scaling

$$V_c \sim I^{1/3} \lambda^{2/3}$$

(13b)

and the position where $T = 0$ in Eq. (8) gives the scaling of the distance from critical to ablation surface

$$x_0 \sim I^{4/3} \lambda^{1/3}.$$ 

(13c)

These quantities then depend only on absorbed irradiance, wavelength, and material.

We now discuss briefly the transformation properties from the ablative flow to laboratory reference frame. One can show that the time dependent fluid equations are invariant to the Galilean transformation

$$v \rightarrow v + V, \quad x \rightarrow x + V, \quad t \rightarrow t - T, \quad Q \rightarrow Q, \quad g \rightarrow g,$$

where $Q$ is the thermal energy flux and $\rho g$ is an inertial force density. Thus, from the steady-state solution in the ablation front reference frame, we can easily generate solutions in the laboratory frame, or any reference frame.

We digress briefly to consider the question of inhibited thermal conduction. First of all, we note that just inside the critical surface, the thermal energy flux is given by $3 \rho_c v_c T$ and the energy flux decreases as one approaches the solid. Thus, just by the nature of the steady state ablative flow with no preheat the electron thermal energy flux is limited to about $3(1 + m/M)$ times its free streaming value, where $m$ and $M$ are, respectively, the electron and ion masses. This value, about 0.05 for carbon, is in the range of but perhaps a little larger than flux limits which have often been quoted.23 We emphasize that this flux limit has nothing to do with any microscopic physical process (for instance magnetic field or instability) which limits the electron thermal energy flux. Rather, it has to do with the macroscopic properties of the fluid flow which is set up, and as such is included in numerical fluid simulations.

Now let us examine how different theories of flux limitation can affect these results. One theory is that the flux is either the classical value or $\alpha \rho T^{3/2}$, whichever is less.21,23,27 However, this is so, the only solutions to Eqs. (7) in the flux limited regime are $\rho, v, w, T$ all equal to constants and the solution becomes indeterminate. In fact, other authors have made this assumption and have found that parameters in the flux limited ablative flow region vary in space only because of the spherical divergence19,20 or the acceleration of the slab.16

Another theory is to state that $K$ in Eqs. (7) is anomalously reduced. These theories at least relate $Q$ to temperature gradient, which is certainly reasonable. Also theories relating $Q$ to a particular instability generally have this feature.28-31 If $K$ is reduced by some factor $f$ (but is still constant), then the equations for fluid quantities can be solved exactly as before. The only difference is that the distance between the critical surface and solid is reduced by this same factor $f$. Thus, if $K$ is reduced, the steady-state length scales are reduced by the same amount so that the thermal energy flux is unchanged.

To summarize, we have shown that the flow has three distinct regions, the accelerated solid, the steady-state ablative flow between solid and critical density, and the subcritical rarefaction wave. All aspects of the steady-state flow (for instance, temperatures, velocities, rates at which the solid is being eaten away) are determined by three parameters, the material, the laser wavelength, and the absorbed laser irradiance. If there is thermal flux limitation, it most likely man-
ifests itself as a reduced separation between critical and solid density.

III. THE ACCELERATED SLAB

In this section, we discuss the accelerated shell. First, we review Kidder's solution for the shell, and then we derive conditions for neglecting the effect of acceleration on the ablative flow. For a steady-state theory to be viable in the slab, the rise time of the laser pulse is assumed to be long compared to the shock wave transit time through the slab. If all preheat is neglected, the solid is assumed to be a degenerate Fermi gas so that

\[ P = [\rho/\rho_0]^{5/3} P_0, \]

(14)

where \( \rho_0 \) is the ambient density and \( P_0 \) is the ambient internal pressure of about 1 Mb (about 10^12 dynes/cm^2). Since the gas is degenerate there is no thermal conduction. Assuming that the kinetic part of the momentum flux is much less than \( P \), the pressure gradient is just balanced by the inertial force so

\[ \frac{dP}{dx} = -\rho g, \]

(15)

where \( g \) is the acceleration of the slab. Integrating Eq. (15) across the slab we find

\[ P_x = -g \int_0^x \rho \, dx = -gM, \]

(16)

where \( P_x \) is the ablation pressure at the surface of the slab given by Eq. (2b) and \( M \) is the total mass of the slab. We reemphasize that \( P_x \) is determined entirely by the material, laser wavelength, and absorbed irradiance. Equation (16) then gives the acceleration in terms of the mass of the slab. If \( x = x_0 \) is the position of zero density at the front of the slab

\[ \frac{\rho}{\rho_0} = \left( \frac{2}{5} \frac{\rho_0 (x_0 - x)}{P_0} \right)^{3/2}, \]

(17)

where in our configuration \( g < 0 \) and \( x_0 - x < 0 \). The upper density boundary of the slab is just determined by the total mass of the slab. To the right of this upper boundary is the region of ablative flow. For our purposes, we regard this transition as a contact discontinuity across which mass, momentum, and energy flux are conserved. The actual nature of this transition is extremely complicated because material goes from a Fermi degenerate gas to a fully ionized plasma. Thus, Eqs. (7) are not able to treat this transition region in either steady or nonsteady flow.

The next question is what effect the acceleration has on the ablative flow region. To account for the acceleration, one adds a term \(- \rho g\) and \(- \rho v g\) to the right-hand sides of Eqs. (2a) and (7a). Thus, crudely speaking, acceleration is negligible if \( \rho g \ll \rho v g \). According to Eq. (8), \( T \) goes from the critical temperature to zero in a distance \( 4kT_{c}^{3/2}/25\rho v \), so that the effect of acceleration on the ablative flow is negligible if

\[ 4kT_{c}^{3/2}/25\rho v < 1, \]

(18)

where \( T_{c} \) is given by Eq. (12) and \( v_c \) is related to \( T_{c} \) by the Mach one condition.

IV. THE EFFECT OF NONUNIFORM ILLUMINATION

A crucial question for laser fusion driven by ablative acceleration is what degree of nonuniformity of illumination can be tolerated. The slab is ultimately accelerated by the total pressure at the ablation surface, \( \rho A v^2 + \rho A T_A \approx \rho A T_A \). We will use the linearized fluid equations to determine the nonuniformity of pressure at the ablation surface in terms of nonuniformity of absorbed laser irradiance at the critical surface. In order to do this within the context of a steady-state theory, it is of course necessary to assume the flow pattern is stable.

The simplest theory,33,34 the so-called "cloudy day" effect assumes that perturbations set up at the critical surface with transverse wave number \( k \) decay exponentially away from the critical surface as \( \exp(-kx) \). The assumption is that the linearized steady-state equation for any fluid quantity \( \rho \) is \( \nabla^2 \rho = 0 \). Neglected in this theory is (1) the fact that the actual linearized fluid equations are much more complicated and have four independent solutions, two which decay and two which grow away from the critical surface; (2) the fact that one must also solve for the relation between perturbed absorbed irradiance and perturbed fluid quantities at the critical surface; and (3) the fact that three boundary conditions are necessary to specify the solution (since the equations are linear, the overall amplitude of the perturbation is arbitrary). In the description of the "cloudy day" effect, it is assumed on physical grounds that only the solution which decays away from the critical surface is relevant. That is, when the decaying solution has reached the ablation surface, no boundary condition is imposed there to generate a "reflected" solution decaying back to the critical surface. Instead, the slab "absorbs" the nonuniformity at the ablation surface by accelerating nonuniformly.

In this section we improve on the "cloudy day" model by (1) actually solving the linearized fluid equations; (2) relating perturbed absorbed irradiance to perturbed absorbed ablation pressure at the critical surface; and (3) imposing reasonable boundary conditions at the perturbed critical surface. However, we make the same physically motivated approximation of considering only solutions which decay away from the critical surface.

To proceed, we will scale the equations so that the dependent and independent variables are \( \theta = \rho/\rho_0, \quad v = v/|v_c|, \quad \tau = T/T_c, \quad \chi = x/x_0, \quad \rho = y/x_0, \quad \kappa = k\chi, \) where \( x_0 = kT_{c}/\rho_0 \). In terms of the scaled variables, the analytic solution for temperature, Eq. (8), becomes

\[ \tau = \left( 1 + \frac{2}{5} \chi \right)^{2/5}, \]

(19)

so that the separation between \( \chi_c \) and \( x_c \) is 0.16. It is now assumed that all quantities are the \( x \) dependent solutions described in Sec. II plus a small transverse perturbation proportional to \( \exp(iky) \). The steady-state fluid equations are then linearized in the small perturbations to give

\[ \frac{d}{d\chi} \bar{S} + \kappa v_c \bar{v}_x = 0, \]

(20)

\[ \frac{d}{d\chi} + \kappa \theta \bar{v}_y = 0, \]

(21)

\[ \theta \bar{v}_x \frac{d}{d\chi} \bar{v}_y - \frac{\kappa \tau}{\tau^2 - \tau^2} (2\bar{v}_x - \bar{S}) = \kappa \theta \bar{v}^2 - \tau, \]

(22)

\[ \frac{d^2 \bar{\tau}}{d\chi^2} + \frac{10 \bar{\theta} \bar{v}_x}{\tau^{5/2}} \frac{d\bar{\tau}}{d\chi} - \kappa^2 \theta \bar{\tau}^2 = \frac{5}{2} \frac{d\bar{\tau}}{d\chi}, \]

(23)
where in writing Eqs. (20)-(23), we have changed notation slightly by redefining \( \dot{v}_x \rightarrow v_y \), so that each equation is real. A tilde superscript indicates a perturbed quantity. Also, we have used as dependent variables \( \tilde{S} \) and \( \tilde{J} \) instead of \( \bar{\theta} \) and \( \dot{\bar{v}}_y \). The quantity \( \tilde{J} \) is the perturbed mass flux in the x direction

\[
\tilde{J} = \dot{\bar{v}}_y + \theta \dot{\bar{v}}_y,
\]

and \( \tilde{S} \) is the perturbed xx component of the total momentum flux tensor (the perturbed ablation pressure)

\[
\tilde{S} = \dot{\bar{v}}_y^2 + \tau + 2 \theta \dot{\bar{v}}_y \bar{v}_y + \theta \dot{\bar{v}}_y.
\]

Equation (20) is the x component of the momentum equation, Eq. (21) is the mass equation, Eq. (22) is the y component of the momentum equation, and Eq. (23) is the temperature equation. In writing the latter, we have made use of Eq. (20) to eliminate several terms. It is precisely \( \tilde{S}_\alpha \) which we want to relate to the nonuniformity in laser irradiance.

The simplest approach\(^{3,3,4}\) is the cloudy day, that is to just assume that all quantities decay away from the critical surface as \( \exp(-k_x \tau) \). However, as we shall see, not only is this a poor approximation, it does not show how to relate the perturbed absorbed irradiance \( \tilde{J} \) to the perturbed ablation pressure \( \tilde{S}_\alpha \) at the critical surface either. Another approximation is to decouple Eqs. (22) and (23). Then, Eqs. (20)-(22) reduce to equations for isothermal sound waves at zero frequency, and Eq. (23) is the equation for temperature perturbations in a flow. The pressure perturbations are carried in by the sound wave, not by the temperature wave. If gradients of equilibrium quantities are neglected, then the sound wave perturbation decays away from the critical surface as \( \exp[K_y \chi] \) (it decays toward negative \( \chi \)). Eqs. (20)-(22) give the dispersion relation

\[
K_y = \kappa(1 - \mathcal{M}^2)^{-1/2}.
\]

The WKB solution would then give

\[
\frac{\tilde{S}_\alpha}{S_c} = \exp\left(-\int_{\chi_c}^{\chi} \kappa(1 - \mathcal{M}^2)^{1/2} d\chi\right)
\]

\[
= \exp(-1.34\kappa(\chi_c - \chi_\alpha)).
\]

(The singularity at \( \mathcal{M} = 1 \) is integrable.)

On the other hand, the temperature perturbation at constant \( \tilde{S} \), \( \tilde{J} \), and \( \dot{\bar{v}}_y \) decays away from the critical surface with

\[
K_T = \frac{5\dot{\bar{v}}_y}{\tau^{1/2}} + \left(\frac{250}{3}\frac{v_y^2}{\tau} + \kappa^2\right).
\]

Notice that unless \( \kappa \) is very large, \( K_T \) is not even approximately equal to \( \kappa \), but is much larger because the fall off in \( \tau \) is dominated by the flow. This is particularly true near the ablation surface where \( \tau < 1 \). We expect the uncoupled solutions to Eqs. (20)-(23) to be approximately valid as long as \( K_T \) and \( K_y \) are far apart. For the \( k \)'s we consider \( (k_x \alpha \leq 20) \), they are far apart near the ablation surface, but not near the critical surface where \( K_y \) increases due to the \( (1 - \mathcal{M}^2)^{-1/2} \) factor.

The procedure we use to solve Eqs. (20)-(23) is as follows. Initialize the variables near the ablation surface where the solutions are nearly uncoupled. The WKB solution to each equation is then valid if \( K_{\tau}^{-2}dK_{\tau}/d\chi \) and \( K_T^{-2}dK_T/d\chi < 1 \). This is easily satisfied for the temperature wave, but not quite as easily for the isothermal sound wave. For instance, if we initialize at \( \tau = 1/4 \), which is very close to the ablation surface, the WKB approximation for the sound wave is valid for \( k_x \alpha \geq 2 \). Then, use the WKB solution for the temperature wave to relate \( \tau_\alpha \) to \( d\tau_\alpha/d\chi \) and the WKB solution for the isothermal sound wave to relate \( \dot{J}_T \) to \( S_\alpha \) and \( \dot{\bar{v}}_y \). \( \tau_\alpha \) will ultimately be related to \( \tilde{J}_\alpha \) by a condition at the critical surface. Then, Eqs. (20)-(23) can be integrated from the ablation surface to the critical surface.

As the boundary condition at the critical surface, we assume that the Mach = 1 condition is satisfied for perturbed quantities also. That is, the connection between the steady-state nonuniform ablation front and the time dependent nonuniform rarefraction wave must still be at \( \mathcal{M} = 1 \). The perturbed critical surface is at

\[
\chi = x_\alpha - \tau / \theta_{\tau}.
\]

The Mach = 1 condition at the perturbed critical surface is then

\[
(\dot{\bar{v}}_x + \dot{\bar{v}}_y)|_{\chi = x_\alpha} = \left[(\tau + \tau_\alpha x_\alpha = x_\alpha)^{1/2}ight]
\]

or

\[
\nu_{\alpha x} \tilde{S} + \nu_{\alpha x} = \nu_{\alpha x} \tau_\alpha x_\alpha + \tau_\alpha x_\alpha.
\]

Relating the primed equilibrium quantities to each other through Eqs. (1a) and (2a), and making use of the fact that \( \nu_{\alpha x} = \tau_\alpha x_\alpha \), it is not difficult to show that Eq. (28b) reduces to

\[
\tilde{J}_\alpha = \frac{1}{2} \tau_\alpha x_\alpha.
\]

Thus, the ratio of \( \tau_\alpha / \tau_{\alpha x} \) is varied until Eq. (29) is satisfied at the critical density. This then specifies the initialization of variables at the ablation surface. Figure 3 shows a plot of \( \tilde{S}_\alpha / S_c \) as a function of \( \kappa \). Also shown by comparison are plots

![FIG. 3. Ratio of the ablation pressure at ablation surface to the ablation pressure at critical surface (dashed). The two solid curves are the cloudy day theory \( \tilde{S}_\alpha / S_c = \exp(-0.16 \kappa) \) and the decay of an isothermal sound wave \( \tilde{S}_\alpha / S_c = \exp(-0.21 \kappa) \).](image)
assuming: (1) all quantities decay as \(\exp(-kx)\) so that \(\frac{S_s}{S_s} = \exp(-0.16k)\), and (2) the pressure decays as an isothermal sound wave decoupled from temperature perturbations so that \(\frac{S_s}{S_t} = \exp(-0.215k)\). The simplest theory shows the least spatial decay of pressure perturbations; the solution to Eqs. (20)–(23), the most.

To completely specify the problem, it is now necessary to derive the relation between \(S_s\) and the perturbed absorbed irradiance \(I\). Equation (12) shows trivially that

\[
\frac{\tau_s}{\tau_e} = \frac{2}{3} I.
\]

Then, making use of the fact that \(S = 2v_s I + \delta \gamma - \gamma_s^2 + \theta \tau_s\), and that \(\gamma_s^2 = \tau_e^c\)

\[
\frac{S_s}{S_s} = 2v_s \tau_e + \theta \tau_s = 2 \frac{S_s}{I} - \frac{4}{3} \frac{S_s}{I},
\]

where we have used Eq. (29) and the fact that \(\nu_e = \theta_e = 1\). This then completely specifies the relation between perturbed absorbed irradiance \(I\) at the critical surface and perturbed ablation pressure \(S_s\) at the ablation surface.

V. APPLICATION TO A LASER PRODUCED PLASMA AND COMPARISONS WITH ONE-DIMENSIONAL FLUID SIMULATION

In this section, we apply the theory developed in the previous sections to laser produced plasmas and compare it with experiment and fluid simulation. If \(T_e\) represents the electron temperature and equipartition is assumed, then the total thermal pressure is \((1 + Z)pT_e / M_i\), where \(M_i\) is the ion mass and \(Z\) is the charge state. The ion mass is the atomic number \(A\) times the proton mass. Then, the quantity \(T\) in Eq. (2), the isothermal sound speed squared is

\[
T = \frac{(1 + Z)T_e}{M_i} \text{ergs.}
\]

From the expression for electron thermal conduction in an unmagnetized plasma,\(^5\) we find that the quantity \(K\) in Eq. (7) is

\[
K = \frac{3.7 \times 10^{-39}A^{7/2}}{(1 + Z)^{7/2}Z_{eq}A'},
\]

where

\[
Z_{eq} = \frac{\sum_a Z^a_{eq} n_a}{\sum_a Z^a_{eq} n_a}
\]

the effective \(Z\) for collisional processes, and \(A\) is the Coulomb logarithm. We now write out the scaling laws for ablation pressure, blow-off velocity, distance from critical to ablation surface, and condition for neglect of acceleration in the ablation front in physical units. The ablation pressure is \(2\rho_s T_e\), assuming Mach one flow at the critical surface. For a CH target used in the Naval Research Laboratory experiments, \(Z = 3.5, A = 6.5, Z_{eq} = 5, \rho_s = 3 \times 10^{-3}\) and we assume \(A' = 5\). In this case, using Eq. (12) to relate temperature to absorbed irradiance, we find

\[
P_a = 2\rho_s T_e = 2.4 \times 10^{12}(I / 10^{13})^{1/3} \lambda^{-2/3} \text{dyne/cm}^2 = 2.4 \times 10^{12}(I / 10^{13})^{1/3} \lambda^{-2/3} \text{Mb},
\]

where \(I\) is the absorbed laser irradiance in W/cm\(^2\) and \(\lambda\) is the wavelength in microns. Notice that there is an advantage

\[
\text{FIG. 4. Comparison of measured ablation pressure to theory.}
\]

\[
\text{FIG. 5. Comparison of measured velocity far from target (solid) with calculated velocity at the critical surface (dashed).}
\]

\[
\text{Manheimer, Colombant, and Gardner 1650 Phys. Fluids, Vol. 25, No. 9, September 1982}
\]

\[
\text{in going to shorter wavelengths. Assuming that the absorbed irradiance is 80% of the incident irradiance, the scaling law given by Eq. (35) is plotted in Fig. 4, along with points taken from the Naval Research Laboratory experiment.\(^5\) Clearly, the agreement is very good, particularly since Eq. (35) is an absolute scaling law with no phenomenological constants. The next quantity of interest is the expansion velocity. The quantity most accessible to theory is the velocity at the critical density which for the plastic target is}
\]

\[
v_c = 2 \times 10^3(I / 10^{13})^{1/3} \lambda^{2/3} \text{cm/sec.}
\]

\[
\text{The quantity most accessible to experimental measurement is the velocity far from the critical surface. The magnitude of this velocity is greater than \(v_c\), but just how much greater depends on how far away the measurement is, and just when the approximation of a one-dimensional isothermal rarefaction wave breaks down (this might be determined, for instance, by the spot size). Figure 5 shows the scaling law given by Eq. (36) as well as measurements from the Naval Research Laboratory experiment.\(^5\) We now consider the distance between the ablation surface and critical surface. According to Eq. (19), this is 0.16 times the distance \(x_0\) where}
\]

\[
x_0 = K T_e^2 / \rho_s = 290(I / 10^{13})^{4/3} \lambda^{14/3} \mu.
\]
At long wavelengths, the scale lengths expand very rapidly. For example, a CO₂ laser produced plasma at 10¹³ W/cm² has a critical to ablation spacing of several meters. Clearly, these long scale length plasma can never form in a laser fusion experiment. Thus, if the thermal transport is classical, the behavior of CO₂ laser produced plasma is dominated by its transient response and will not reach steady state. However, if the transport is inhibited, then the length scales can be reduced. Thus, while short wavelength has the advantage of increasing ablation pressure at a given irradiance, the long wavelength has the advantage of increasing the critical to ablation surface separation so that nonuniformities in laser irradiance can be smoothed out before they reach the ablation surface. This was also recently discovered by numerical simulation.

The next question is the condition for the neglect of the acceleration on the blow-off plasma. Equation (18) reduces to

$$10^{-17} (I/10^{13})^{2/3} \lambda^{10/3} \ll 1.$$  \hspace{1cm} (38)

For accelerations of order 3 × 10¹⁵ cm/sec² as measured in Refs. 1 and 2, the effect on the blow-off plasma is negligible up to irradiances of order 10¹⁵ W/cm² and higher.

We now discuss experiments with non-uniform laser illumination. In Ref. 14 a portion of the laser light was masked out causing an intensity minimum at the center of the laser spot. The ratio intensity at the dip to maximum intensity was 1:2, 1:6, and 1:10 and the wavelength (peak to peak separations) took on values of 280 μm or 440 μm. The velocity nonuniformity of the accelerated target is then measured as a function of irradiance. Here, we compare the results of that experiment to the linear theory developed in Sec. IV. Since that theory is linear in nonuniformity, we compare only with the experiment having the 2:1 irradiance ratio. This experiment had a 280 μm transverse wavelength.

The velocity fluctuation should be proportional to the fluctuation in pressure at the ablation surface times the pulse time, therefore,

$$v_{\text{max}} - v_{\text{min}} = \frac{2S_c}{S_A},$$  \hspace{1cm} (39)

$$\frac{v_{\text{max}}}{v_{\text{min}}} = \frac{2S_c}{S_A}.$$  

The quantity \(S_c\) is related to \(S_A\) by the graph in Fig. 6 and \(S_c\) is related to \(I\) by Eq. (31). Making use of the fact that \(x_0\) is given by Eq. (37), we find that \(v_{\text{max}}/v_{\text{min}} = 1\) versus irradiance is given by Fig. 6. Also shown in Fig. 6 are the experimental points taken from Ref. 10. Clearly, there is very reasonable agreement.

As a check on some of the one-dimensional theory derived here, we have also performed a number of one-dimensional fluid simulations in planar geometry. The code, described elsewhere is modified only by eliminating inverse bremsstrahlung absorption and by depositing all absorbed laser energy at the critical surface. This is in agreement with the theoretical model described in Sec. II. To test the scaling of parameters on wavelength and irradiance for a plastic target, we have performed five simulations with the parameters given in Table I. The parameters are as given for the CH target except that \(x_{\text{off}}\) was taken as 3.5 instead of 5. According to Eq. (37), this would make \(x_0\) of the simulation larger by a factor 10/7. First, we point out that all simulations in the table did come to a steady state in that the separation between critical surface and ablation surface did approach a constant value, with both surfaces moving into the accelerated slab. The third column of the table shows the isothermal Mach number at the critical density in the reference frame of the ablation front. Clearly this Mach number is very near unity. The worst agreement was in the second row where the Mach number was very difficult to measure due to large acceleration of the slab. Approximate power law formulas for the ablation pressure and critical to ablation surface separation are

$$P_A \sim I^{0.6} \lambda^{0.76}, \quad x_0 \sim I^{1.3} \lambda^{4.3}.$$  

These formulas have scaling very nearly as given in Eqs. (35) and (37). Thus, the theory presented here is in reasonable agreement with both experiments and fluid simulation.

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