A Primer on Roller Coaster Dynamics

Part I - Plane and Fancy

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A Primer on Roller Coaster Dynamics, Part II, will appear in the next issue of ROLLERCOASTER! - ED

Is a Coaster Ride Just Some F(t)?

Until recently, I have kept my interest in roller coasters and my professional interest in mechanical engineering in well-defined and separate mental compartments. Occasionally, I've briefly connected the two by using roller coaster examples in the dynamics course I sometimes teach to sophomores at the University of Rochester. Last fall, a student came up after class to ask about a simple force balance I had presented for a passenger going through a loop. He asked me which of the three forces in the balance is actually felt by the passenger. The question made me realize that, from the point of view of dynamics, a roller coaster ride is nothing more than a time history of forces - all that excitement is just some F(t)! After thinking about this awhile, I couldn't resist analyzing the passenger experience in a coaster ride. The result is this series of articles, in which we use elementary dynamics to show how the geometry of a roller coaster and the passenger experience are related. (Some earlier interesting discussions of this sort have been given by Walker [1], Honig [2], and Rathjen [3].)

Of course, most coaster enthusiasts are usually satisfied with less technical descriptions of the coaster experience. One student, interviewed by Robert Cartmell, said he rode coasters "Because a coaster run makes my blood hurt!" [4, p. 88] Not a bad description!

There are also any number of psychological factors which, while leaving accelerometers unaffected, contribute to the "hurting of blood." These can range from unexpected geometric features, to the warning signs at the entrance that suggest an outcome of death for the passenger in less than perfect condition. As interesting and important as these factors are, we won't consider them further here. We'll concentrate instead on the relation between the geometry and the forces experienced.

Our task requires writing equations. Yes, equations! You don't need to analyze a coaster to enjoy it, but you can greatly enhance your understanding of what's happening by mastering a few basic quantitative relations connecting speed, track curvature, and forces. We will first develop the basic kinematical and dynamical relations (for reference material, see for example [5]). Then we will deal with plane hills. We will discuss the conditions necessary for weightlessness at hill tops, and we'll analyze the large forces at hill bottoms. We will develop the relation between speed and drop by using conservation of energy. Finally, we'll look at vertical plane loops, and show, among other things, why such loops are never circular.

The Inertial Force is With You

We begin with a basic analysis of the motion of a passenger in a coaster car. To describe these motions, we need a frame of reference that remains fixed. This is the (xyz) frame shown in Figure 1. The point where the x, y, and z axes meet is called the origin of the frame. We can describe the location of the coaster car at any time by giving its direction and its distance from the origin. In mechanics, we express this information concisely in the form of a vector $\mathbf{r}_c$, as shown in Figure 1. We also need to describe the position of the passenger, which is another vector $\mathbf{r}_p$. Finally, it is convenient to consider also the position of the passenger relative to the car, and this we call $\mathbf{r}_{p/c}$. These three vectors form a triangle, so that

$$\mathbf{r}_p = \mathbf{r}_c + \mathbf{r}_{p/c}. \qquad (1)$$

To talk about the forces on the passenger, we are going to use Newton's second law, which relates the forces to the accelerations. The accelerations involved are the acceleration of the car, $\mathbf{a}_c$, the acceleration of the passenger, $\mathbf{a}_p$, and the acceleration of the passenger relative to the car, $\mathbf{a}_{p/c}$. By differentiating equation (1) twice with respect to time, we get a basic relation between these three acceleration:

$$\mathbf{a}_p = \mathbf{a}_c + \mathbf{a}_{p/c}. \qquad (2)$$

According to Newton's second law, the sum of the forces on the passenger is equal to the mass m of the passenger times the acceleration $\mathbf{a}_p$ of the passenger. The forces acting on the passenger are the gravity force mg (where g is the acceleration of gravity), and the seat force S. This seat force S is the total force exerted on the passenger by the coaster car, through contact with the bottom or back of the seat, through the contact of the feet with the floor, through the white knuckles on the lapbar, or perhaps through a hapless companion pinned to the side of the coaster car. Then we get from Newton's law and equation (2) the following equation:

$$\mathbf{S} + m\mathbf{g} = m(\mathbf{a}_c + \mathbf{a}_{p/c}). \qquad (3)$$

At this point, let's make a simplifying assumption. Let's ignore $\mathbf{a}_{p/c}$, the acceleration of the passenger relative to the car. At first thought this seems wrong, because it precludes describing the event of being thrown violently about in the coaster car, which is certainly a typical experience. However, in most coaster rides, there are relatively long intervals in which the passenger is pressed into a fixed position relative to the car. These long intervals are broken by shorter violent shiftings, which occur whenever a change in track curvature requires a component of the seat force to change sign. Remember that the contact forces are always compressive, so that, for example, a change in sign of the horizontal seat force requires a physical shift of the passenger from one side of the compartment to the other. By ignoring the relative acceleration, we give

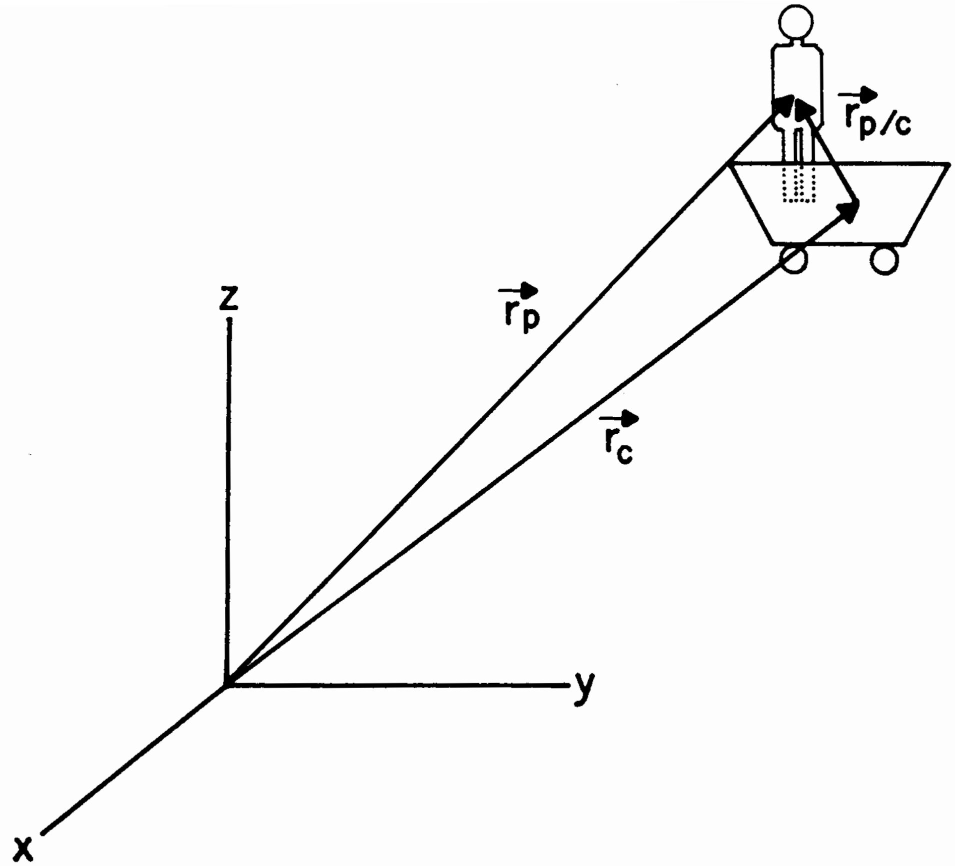


Figure 1. Position vector of passenger, rp, and coaster car r_c, in fixed xyz reference frame. Position vector, $r_{p/c}$, of passenger relative to car.

up the ability to describe these short violent events, but we can still describe the typical longer intervals. (In many of the newer coasters, the seat, lapbar, and harness configuration is sufficiently tight that the relative acceleration is in fact always small.) In the absence of $\mathbf{a}_{p/c}$, the basic equation can be written as

$$S + mg - ma_c = 0. \qquad (4)$$

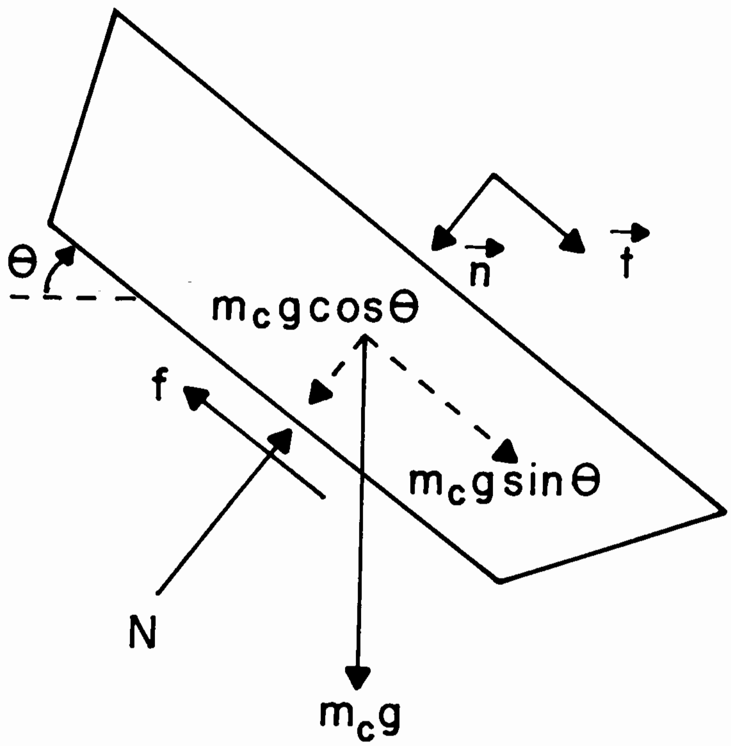
Equation (4) is the equation on which all the rest of our analysis is based. It allows us to calculate the seat force in terms of gravity g, the passenger mass m, and the coaster acceleration $\mathbf{a}_c$. Because this equation is so fundamental, let's take a closer look at it. (You mechanics experts will recognize it as a form of d'Alembert's principle, sometimes called the equation of dynamic equilibrium.) The equation says that the passenger is in equilibrium under the action of three forces: the seat force S, the gravity force mg, and an inertial force ($-ma_c$) associated with the acceleration of the coaster car. (A familiar special case of the inertial force is the centrifugal force associated with motion in a curved path.) This is the equation I mentioned earlier, about which my student asked, "Which forces does the passenger feel?" That is actually a good question. Even though the forces add up to zero, you feel **all** of them. The seat force S is a distributed surface force, which you feel at every point of your contact with the coaster car. The gravity force mg is just the familiar body force that always acts on every part of your body. The inertial force ($-ma_c$) is also a body force like gravity. Sometimes students of mechanics think of the inertial force as a mathematical convenience, but not really a force. This view would never be adopted by coaster riders. We know better. We know that **the inertial force is a real force - just as real as gravity**. Equation (4) shows that quite clearly, because the inertial force enters in **exactly** the same way as the gravity force. You don't believe that? Then ride a loop coaster. As you enter the loop, even those of you under 30 will have a double chin, courtesy of the inertial force, which at that point acts like an extra large gravity force.

One of the most exciting possibilities in any coaster ride falls very simply out of equation (4). Suppose the coaster acceleration $\mathbf{a}_c$ is equal to the acceleration of gravity g. Then equation (4) says that $S = 0$. What is that state? It is the wonderful Nirvana of weightlessness! If we can make $a_c = g$, then we can all float like astronauts, if only for a moment. Of course you can just drop the coaster car from a tower to achieve this, but passenger survival is after all a standard constraint in the entertainment industry. Actually there are some tower rides that do have a short free fall, and there are now even coaster-like rides with an initial free fall, such as the *Demon Drop* at Cedar Point. But can we achieve weightlessness with a more-or-less traditional coaster? Let's try to answer that question by looking at some simple hill configurations in which the track is all in a single vertical plane.

Do You Really Come Out of Your Seat on That First Drop?

Many coaster riders tell stories about first hills so steep that the passengers are weightless, floating out of their seats. This folklore suggests that we start our search for weightlessness by looking at a coaster car on a long, steep, straight hill. We consider a car of mass m_c on a hill with slope angle θ. Figure 2 shows the forces acting on such a car. In analyzing such forces, we have to introduce two reference directions. The first is the tangent direction, which is the direction parallel to the track in the direction of motion, and the second is the normal direction, which is perpendicular to the track. In mechanics, we specify such directions in terms of unit vectors pointing in those directions. You can think of these unit vectors as one-foot rulers mounted on the coaster car. The unit tangent vector t points in the direction of motion, and the unit normal vector **n** is perpendicular to the floor of the car. Then the gravity vector g can be resolved into a component $g\cos(\theta)$ in the normal direction **n**, and a component $g\sin(\theta)$ in the tangential direction **t**. In addition to the gravity force,

Figure 2. Free body diagram of a coaster car on a straight hill with a constant slope angle θ. The unit vectors are the track tangent t in the direction of the motion, and n normal to the track. The forces are gravity, resolved into a component $m_c g\sin(\theta)$ along t and a component $m_c g\cos(\theta)$ along n, the normal track force N, and a force f which includes friction, and the coupling force from other cars in the train.



there is a normal force N exerted by the track, and a force f in the positive or negative t direction. The force f in the direction of the motion can be caused by friction between the track and car, by air resistance, or by coupling forces between the cars in a train. In the actual design of a coaster, it is important to include the effects of friction and air resistance. (Without these forces, coasters would return to the station at 60 miles per hour rather than 5 or 10 miles per hour.) For our more limited goals, however, we'll sacrifice some accuracy for simplicity, and ignore the friction and air resistance forces here. The third origin of f, the coupling force between cars, gives rise to some important and interesting effects which we will consider in detail in Part III of this Primer. For now, however, we will keep the discussion simple by considering only a single coaster car. Thus we take f = 0 from here on.

To simplify our notation, we denote the acceleration of the car by **a**, rather than the notation a_c used earlier. The component a_n of the acceleration in the normal direction **n** is zero because the track is not curved. The acceleration a_t in the tangential direction t is obtained from Newton's law:

$$m_c a_t = m_c g\sin(\theta) \quad ,$$

so

$$a_t = g\sin(\theta) \quad ,$$

and hence the vector acceleration is (remember $a_n = 0$)

$$\mathbf{a} = a_t \mathbf{t} = g\sin(\theta)\mathbf{t}. \tag{5}$$

Now we consider the basic force balance on the passenger, given by equation (4). It becomes

$$S + m[g - g\sin(\theta)t] = 0.$$

The resolution of g into tangential and normal components is

$$g = g\cos(\theta)\mathbf{n} + g\sin(\theta)t \quad , \tag{6}$$

and by substituting this into the previous equation, we get

$$S + mg\cos(\theta)\mathbf{n} = 0. \tag{7}$$

Now let's see where all this analysis has gotten us. Consider for a moment a passenger sitting in a level coaster car at rest. For such a passenger, the gravity force is down, the seat force is up, and they just balance — in more technical terms, S + mg = 0. Comparing this with equation (7), we see that the passenger on the hill feels a gravity pointing in the **normal** direction - that is, down for the passenger is perpendicular to the floor of the car rather than in the direction of g. Furthermore, the magnitude of the gravity felt by the passenger is less - it is $g\cos(\theta)$. Thus the passenger's weight is reduced by the factor $\cos(\theta)$. The effect can be considerable. For example, the *Cyclone* at Coney Island has a first hill with a slope angle of 60 degrees [6, p. 50], which gives $g\cos(\theta)$ = 0.5g, thus reducing gravity by one-half. An even more extreme case is the Intamin Shuttle Loop which has a 72-degree hill [4, p. 45], giving $g\cos(\theta)$ = 0.31g. Although these are substantial reductions in g, the states are still far from weightlessness. How can we account for the folklore of the first hill which claims weightlessness? Partly it is just the reduction from full gravity to half or even one-third gravity, not an everyday experience for most of us. The friction force on the coaster car, which we neglected in our analysis, also can contribute. The effect of the friction force is the same as putting on the brakes in a car; it tends to make the passenger move forward relative to the car, contributing to the feeling of weightlessness. Another contributing factor on a curved hill is related to the first car, last car difference which we will discuss in Part III. Of course, as always, the wind in the face and the screams in the ears augment the dynamical effects.

Curvature is Anti-Gravity

Let's continue our pursuit of weightlessness. At this point, we see that it can't be achieved with a straight hill. Motion on any straight hill is a kind of partial free fall, and we can't cancel gravity entirely unless the hill is straight down. Another way to produce an acceleration, however, is to use a curved track which will give us a centrifugal force. Figure 3 shows the geometry for a coaster on a plane, but curved hill. We assume that the center of curvature is below the track, so that the unit normal **n** points toward the center of curvature. A new quantity enters the analysis here, namely the radius of curvature R. If V is the coaster speed, then the normal acceleration is $(V^2/R)\mathbf{n}$ (this is just a fancy version of the familiar centrifugal force). The tangential acceleration is, as before, $g\sin(\theta)t$ (we are, as before, neglecting the friction force). The total vector acceleration is the vector sum of these. Substituting that sum in equation (4) and using equation (6) for g, we get

$$S + m[g\cos(\theta) - (V^2/R)]\mathbf{n} = 0 \quad , \tag{8}$$

or $S = -m[g\cos(\theta) - (V^2/R)]\mathbf{n}$.

This rather complicated looking equation contains the key to weightlessness. To be weightless, a passenger must have a zero seat force, and according to equation (8), this will happen whenever the inertial force is equal to the component of gravity in the normal direction — that is, whenever $(V^2/R) = g\cos(\theta)$. Thus curvature can cancel gravity completely if the speed is right. How much speed and how much curvature? It is surprisingly easy. Consider a coaster at the top of a curved hill, as shown in Figure 3. At such a point $\theta = 0$, so $\cos(\theta) = 1$ and we need $V^2 = gR$ to become weightless. Table 1 below gives a few combinations of V and R values which do this for us. (Just a note for those of you who might not have done this kind of calculation for a few years. You must use a consistent set of units. Here we use feet (ft) for lengths, seconds (s) for time, and feet per second (ft/s) for velocities. Thus miles per hour must be converted to ft/s by using 1 mile per hour = 1.47 ft/s. We also need the value of g, which is 32.2 ft/s^2.) It is clear from

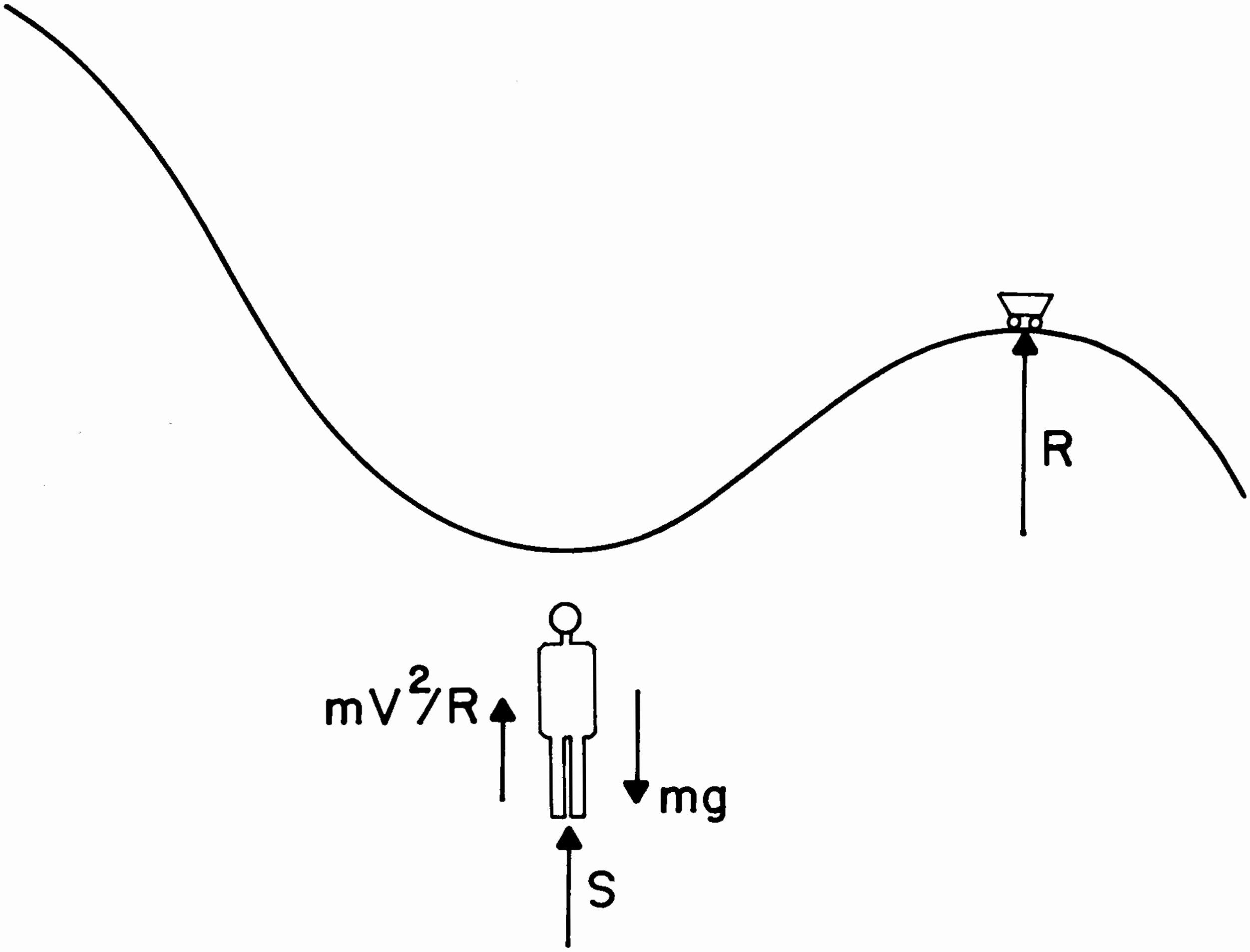


Figure 3. Forces at a hill top on a passenger of mass m: the seat force S, the downward gravity force mg, and the upward centrifugal force m^{V2}/R, where V is the speed and R is the radius of curvature. The gravity and the centrifugal forces are in the opposite direction and the set force is the difference of the two.

Speed (miles per hour)	Radius (ft)
20	27
30	60
40	107
50	167
60	240

Table 1. Values of speed and radius of curvature which give weightlessness at the top of a hill.

the table that the requirements for weightlessness are modest, and they are in fact met by many real coasters. By using even larger values of V or smaller values of R, we can effectively turn gravity upward. In that case, the seat force S would actually be downward, and would be supplied by a lap bar or a shoulder harness. This situation occurs in real coasters. The *Racer*, a coaster at Kings Island designed by John Allen, actually lifts off the track for 140 feet [4, p. 59]. (Sometimes you can spot such segments of track because they are not shiny.) The double wheels (above and below the track) keep the car on the track, and the lapbar keeps the passenger in the car.

What Goes Up Must Come Down - Fast

The most common configuration used to produce weightlessness is a drop down a large hill (often the first) to provide speed, followed by one or more smaller, sharply curved hills. How high does the large hill have to be to give the speed V that we need? More generally, what is the relation between drop and speed? If, for example, a park advertises a 90 mile per hour coaster and a 100-ft first hill, should we believe them? We can answer such questions by invoking conservation of energy. In the absence of friction, the sum of the kinetic energy and the gravitational potential energy of the coaster will be constant. At the top of the highest hill, the velocity is generally negligibly small. At the bottom of the drop, the velocity V_B will be a maximum. The potential energy at the top of the hill is $m_c gH$, where H is the height, and the kinetic energy at the bottom is $(m_c/2)V_B^2$. By equating these, we get

$$V_B = (2gH)^{1/2}. \quad (9)$$

This value of V_B is actually an upper bound because of friction, but it is not a bad estimate. Table 2 shows some values of V_B for some well-known coasters. We see that a top speed of around 60 miles per hour is typical. Because of the square-root dependence of speed on drop, it's difficult to get appreciably larger speeds without enormous increases in height, with attendant cost and structural problems. For example, to get 90 miles per hour, we need a drop of over 270 ft.!

Coaster	First Drop (ft)	Speed (miles per hour)
Thunderbolt (Kennywood)	90	52
Texas Cyclone (Astroworld)	92	52
Comet (Crystal Beach)	96	54
Great American Scream Machine (Six Flags Over Georgia)	105	56
Racer (Mexico City)	110	57
Beast (Kings Island)	135	64

Table 2. Speed and drop for several coasters (drop data from [4]).

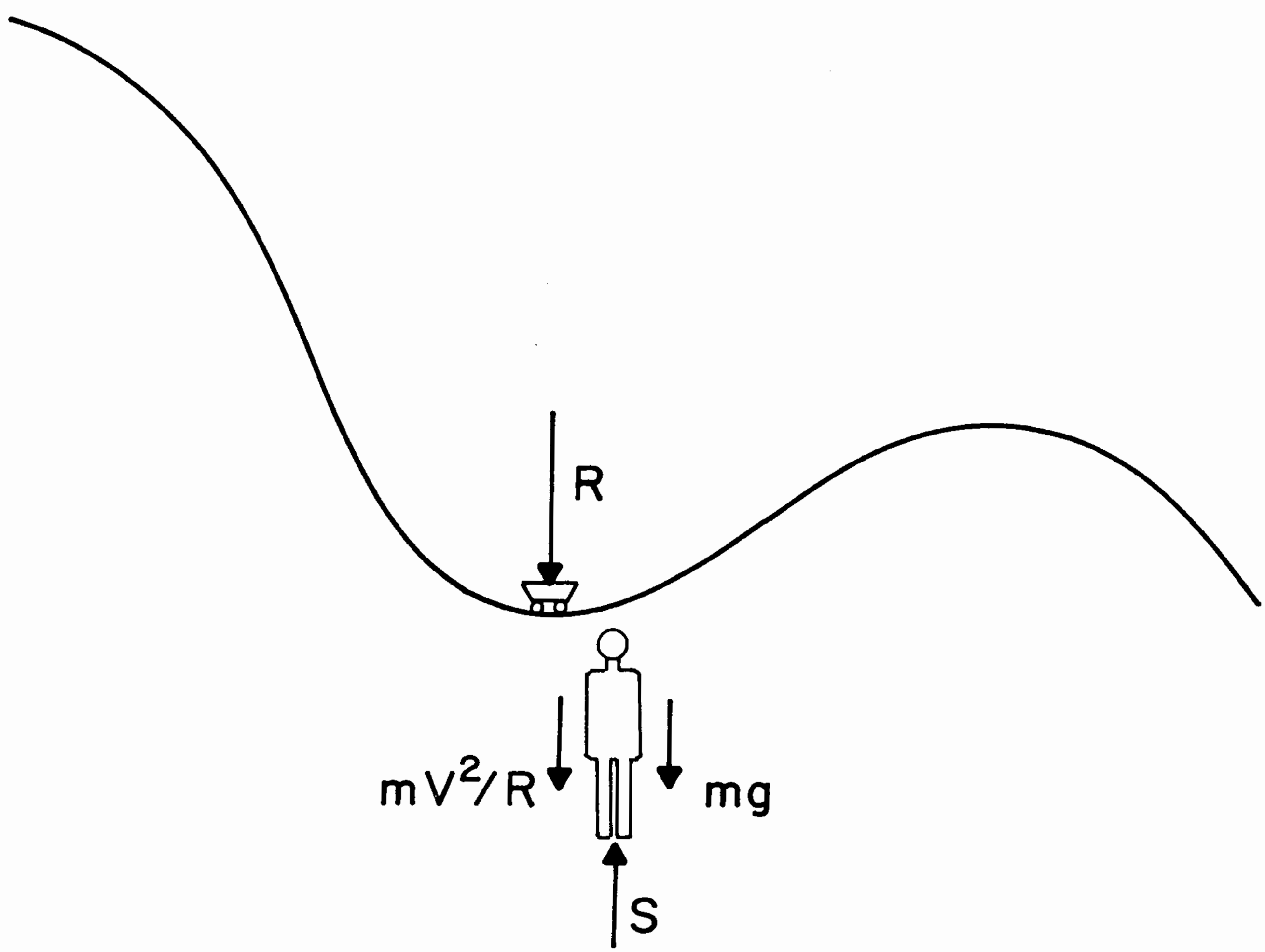
While we need speed to produce weightlessness, the speed can actually be a problem at the bottom of a hill. Consider the configuration shown in Figure 4. At the bottom of the hill, both the inertial force and the gravity force are downward. The seat force must balance them both, and hence is upward and has a magnitude

$$S = m[g + (V^2/R)]. \quad (10)$$

Table 3 shows the total seat force as a function of radius of curvature, for a speed of 60 miles per hour. For a sharply curved hill with a radius of 25 ft, we get over 10 g's! Even a well-trained fighter pilot would be taxed by that. To make the ride tolerable to those of us without training and without anti-g suits, the radius of curvature of the track has to be increased so that the seat force is in an acceptable range.

What is an acceptable range? That question doesn't have a simple answer. Large g's can be tolerated for short times. For example, blackout occurs in about 4 seconds for 10 g's, 5 seconds for 8 g's, 6 seconds for 6 g's, 8 seconds for 4 g's, and generally not at all below 4 g's [7, p. 547]. Thus one really has to calculate the time course of the g's in the design of the coaster. That is not difficult, but it would involve more detail than is appropriate for our discussion.

Figure 4. Forces at a hill bottom on a passenger of mass m: the seat force S, the downward gravity force mg, and the downward centrifugal force mV^2/R, where V is the speed and R is the radius of curvature. The gravity and the centrifugal forces are in the same direction and the seat force is the sum of the two.



Radius (ft)	Seat Force (in g)
25	10.6
50	5.8
75	4.2
100	3.4
125	2.9
150	2.6

Table 3. Total seat force (as a multiple of g) as a function of the radius of curvature at the bottom of a hill when the speed is 60 miles per hour.

I have seen the claim that some real coasters produce seat forces as large as 6 g's for short times, but I haven't seen detailed enough data to verify that by analysis. Certainly the parameters necessary to produce 6 g's are accessible, as we can see from Table 3.

So far, we have been in pursuit of weightlessness. There is another condition that is equally appealing to most coaster riders, and that is being upside down. Let's turn now to loop coasters, and see what dynamics can tell us about them.

Goodbye and Good Luck

It seems that roller coasters induce a three-tiered order in the set of all humans. There are those who won't ride any coasters, there are those who will ride any non-looping coasters, and then there are those who will ride anything. Somehow I made the transition to the third set with my daughter not too many years ago on the *SooperDooperLooper* at Hershey Park. I still remember distinctly the words of the station attendant as the train rolled out of the station - "Goodbye and good luck!"

I could claim that there is no luck to it and that my knowledge of mechanics gives me a perfectly rational view of the safety of a loop coaster ride, but that wouldn't account for that wonderful sinking feeling in the pit of my stomach as I approached my first loop. Who knows why we are willing to get on a machine that will turn us upside down at high speed? I guess the hurting blood answer is as good as any. Or perhaps Mallory's famous answer applies to the small challenge of a loop coaster as well as to the monumental challenge of Everest: we ride it because it is there.

Enough philosophy. Let's see what it takes to keep you in your seat at the top of a loop. Consider a circular loop of height H, as shown in Figure 5. Suppose the speed at the top is V_T. The radius of curvature is H/2. Then the seat force S_T at the top of the loop is

$$S_T + m[g - (2V_T^2/H)]\mathbf{n} = O. \qquad (11)$$

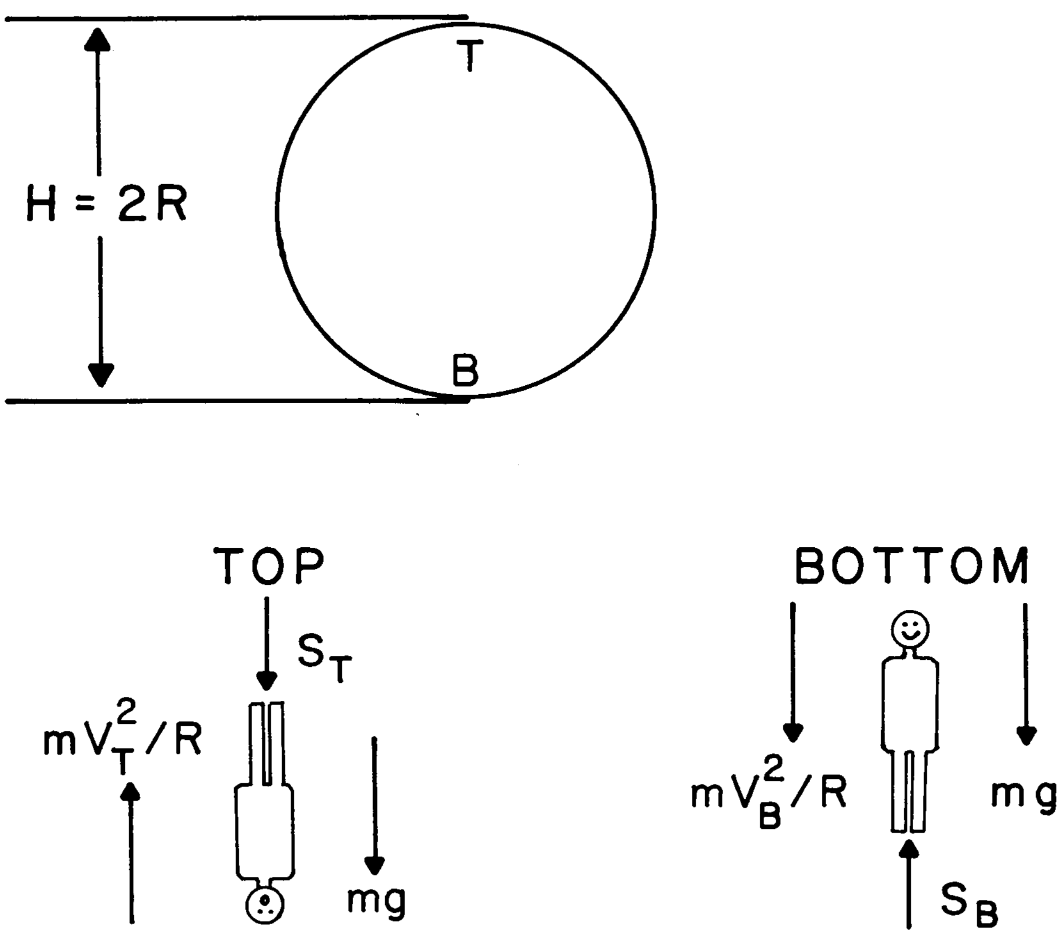


Figure 5. Forces on a passenger of mass m at the top and bottom of a circular loop of radius R. The speed at the top of the loop is V_T, and the speed at the bottom of the loop is V_B. At the top, the gravity force mg and the centrifugal force mV^2_T/R are in opposite directions, and the seat force, which is the difference of the two, is small. At the bottom, the gravity force mg and the centrifugal force mV^2_B are in the same direction, and the seat force, which is the sum of the two, is large. The effect is augmented by the fact that V_B is larger than V_T by an amount which can be calculated from conservation of energy.

At the top of the loop, **n** is pointing straight down, in the direction of gravity. Loop coasters usually are designed so that there is a positive contact force with the seat, even at the top of the loop. They all have shoulder harnesses, so that even if a wheel bearing fails and speed drops below the design value, you won't fall out. But in a normally functioning loop coaster, the harness is redundant. To calculate the speed required at the top of the loop, suppose that the contact seat force — which is down because you are upside down — is 1/4 of your normal weight. That is, we take $S_T = (1/4)mgn$. Then equation (11) gives us

$$V_T^2 = 5gH/8. \qquad (12)$$

If, for example, the loop is 60 ft high, then we get $V_T = 24$ miles per hour, a modest speed. However, there is still a problem. Consider what happens at the bottom of the hill. We can calculate V_B, the speed at the bottom, by using conservation of energy. The kinetic energy at the bottom $((m_c/2)V_B^2)$ is equal to the kinetic energy plus the potential energy at the top $((m_c/2)V_T^2 + m_c gH)$. Thus we get

$$V_B^2 = V_T^2 + 2gh. \qquad (13)$$

The seat force S_B at the bottom of the hill then may be obtained from equations (10), (11) and (13), and we find that S_B is upward, and has magnitude

$$S_B = m[g + V_B^2/R] = S_T + 6mg. \qquad (14)$$

In order to get an adequate seat force at the top, we must endure a seat force at the bottom which is larger by 6 g's - a bit much for most passengers! Thus conservation of energy provides a very tough constraint on design. How do we get around this problem? We can't change conservation of energy, but we can change the geometry. Real loop coasters are not circular. (Figure 6 shows a typical loop shape.) The upper part of the loop is sharply curved and circular. The lower part, however, is a curve along which the radius of curvature continuously increases as you descend. The main point is that the radius of curvature is varied so that it is large where the speed is large. This keeps the centrifugal force mV^2/R within reasonable bounds.

Now let's apply our newly acquired coaster analysis skills to two common types of shuttle loop coasters. We begin with a horizontal reverse point coaster. Figure 7 shows a typical example. This coaster leaves the station, goes forward through the loop, climbs the hill at the far side, reverses, returns backwards through the loop and arrives back at the loading station. This configuration presents an interesting design problem, because the train must return to the same height at which it starts without benefit of a powered hill climb. In our idealized analysis without friction, this is easy, but in the real world it is impossible - the coaster must acquire some additional energy to make it back. This is accomplished by catapulting the coaster out of the station, thus giving it an initial kinetic energy sufficient to compensate for the frictional losses.

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Figure 6. Loop geometry in a typical loop, **Vortex**, Kings Island, Kings Island, Ohio.

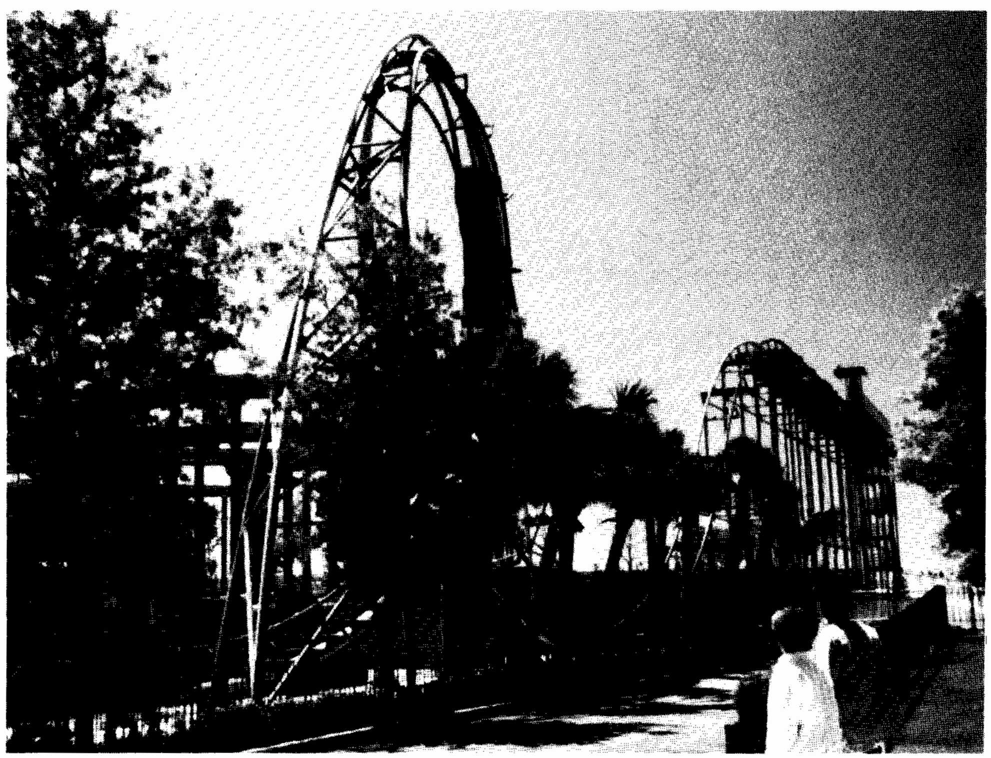

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Figure 7. A shuttle looper with horizontal reverse points, the **Double-Oh**, Boardwalk and Baseball, Orlando, Florida.

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Figure 8. A shuttle looper with vertical reverse points, **White Lightnin'**, Carowinds, Charlotte, North Carolina.

Consider some typical numbers for such a coaster [6, p. 11-12]. The station is about 50 ft high, the loop diameter is about 30 ft, and the maximum speed is 45 miles per hour.

We can easily assess the effect of the catapult. A drop of 50 ft from rest would give a speed of only 39 miles per hour, corresponding to 25% less kinetic energy than for 45 miles per hour. We are also told that passengers experience about 3.5 gs at the bottom of the loop [6, p. 11-12]. If the loop were circular, we could use the above numbers and equation (9) to estimate a seat force of over 10 gs. From the given 3.5 gs and the given speed of 45 miles per hour, we can use equation (9) to estimate that the radius of curvature at the bottom of the loop is $R = 54$ ft.

Now let's look at a shuttle looper with vertical reverse points. Figure 8 shows a typical example. Consider the passenger experience, with some typical numbers [6, p. 11-12]. The train is launched from the station, which is the lowest point of the track, by a catapult. It reaches a speed of 55 miles per hour in only 4.2 s, corresponding to a horizontal acceleration of 0.6g. After going through a 60-foot loop, the train climbs a 140-ft hill that reaches an angle of 70 degrees. Riders near the front clearly see the end of the track approaching while they are still moving. The train then falls by gravity through the loop backwards, through the station at high speed, and up a 112-foot hill at the other end. Finally, the train falls down this hill and is braked to a stop in the station. How long for all of this? A mere 37 seconds! Definitely a blood-hurter. Here's how you can one-up your fellow passengers as the train nears its high point on the 142-ft hill. While they are in a panic at the thought that the train may actually go off the end of the track, you can announce casually that conservation of energy guarantees that a train traveling at a speed $V = 55$ miles per hour $= 80.7$ ft/s cannot possibly rise any higher than $H = V^2/(2g) = 101$ ft.

Another interesting variation on the looper has appeared recently at Kings Island. This is *King Cobra*, a loop coaster which the passengers ride standing up. You faint and you're dead! Well, not really. There is an elaborate shoulder harness and waist bar. At the top of the loop, you will notice that your knees are bent. This can't be due to fear for an ACE member, so it must be the centrifugal force. If you have followed our discussion so far, then you will be able to show that the bent-knees observation allows the conclusion that the centrifugal force at the top of the loop must exceed 2 g's.

The Track Ahead

So far, we've looked at a number of coaster configurations which have one thing in common - they are all in a single vertical plane. Many interesting coasters don't fit this category. In Part II, we'll talk about some fully 3-dimensional geometries associated with corkscrew coasters, beltless coasters (such as the *Speedracer* at Worlds of Fun), suspended coasters, and runaway mine trains. The main new feature in Part II will be a discussion of car orientation and the banking of the coaster track.

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