Problem #1 – 10pts: Looking at Appendix A Eq. A.15, for \( \{x\} = [x, x, \ldots, x] \) and \( \varphi = \frac{1}{2} \) \( \{x\}^T[A]\{x\} \) with \( [A] \) being independent of \( x \).

a. Show the following for \( [A] \) being an arbitrary \( n \times n \) matrix

\[
\frac{\partial \varphi}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial \varphi}{\partial x_1} & \frac{\partial \varphi}{\partial x_2} & \cdots & \frac{\partial \varphi}{\partial x_n} \end{bmatrix}^T = \frac{1}{2} (\{A\} + [A]^T)\{x\}
\]

b. Show if \( [A] \) is an arbitrary and symmetric \( n \times n \) matrix

\[
\frac{\partial \varphi}{\partial \mathbf{x}} = [A]\{x\} \quad \text{and} \quad \frac{\partial^2 \varphi}{\partial x_i \partial x_j} = A_{ij} = A_{ji}
\]

Problem #2 – 7pts:

8.2-1 Let \( [E'] \) be 3 by 3, as for a plane stress problem. Show that Eq. 8.2-10 yields \( [E'] = [E] \) if the material is isotropic.

\[
[E] = [T_\varepsilon]^T[E'][T_\varepsilon] \quad (8.2-10)
\]

Problem #3 – 10pts: Determine the magnitude and location of the maximum deflection od the beam AB in terms of \( b, L, E, I \) and \( P \). You may assume that \( a > b \).
**Problem #4 – 10pts:** A body is experiencing a variable stress state as shown where \( X_i \) indicate coordinates in 3D space. Comment if the body is in static equilibrium and if not, what balancing force is required to establish equilibrium. Here, \( B \) and \( b \) are constants.

\[
[\sigma] = B \begin{bmatrix}
X_1^2 X_2 & (b^2 - X_2^2) X_1 & 0 \\
(b^2 - X_2^2) X_1 & \frac{1}{3} (X_2^2 - 3b^2) X_2 & 0 \\
0 & 0 & 2bX_2^2
\end{bmatrix}
\]

**Problem #5 – 6pts:** For the infinitely long, half cylinder body with radius \( a \), as shown below where \( X_1 \) and \( X_2 \) indicate coordinate bases, provide a complete statement of all the boundary conditions in 3D. The body is exposed to a uniform traction load \( q \) pointing downward on the top surface.

Note: Do not attempt to solve the system for unknown stress and strain fields.

**Problem #6 – 12pts:**

The plane structures shown consist of rigid weightless bars connected by linear springs with stiffness \( k \). Degrees of freedom are horizontal translation \( \theta_i \) for \( i = 1, 2 \) as shown. Vertical motion and out of plane displacements are not allowed. Determine the structural stiffness matrix for each case by enforcing equilibrium.

Hint: Linear elasticity follows superposition principal. So you can separate degrees of freedom.