Problem #1 – 5pts:

2.3-4  To elevate the end of a cantilever beam without rotating it, as shown, force and moment are required. From the information shown, fill in as many numerical values as you can in an element stiffness matrix that operates on nodal d.o.f. \( \{ \mathbf{d} \} = [v_1 \ \theta_{z1} \ v_2 \ \theta_{z2}]^T \), where \( v_1 \) and \( v_2 \) are measured in millimeters. Do not use beam deflection formulas or Eq. 2.3-5. Instead rely on the given data, physical argument, statics, and the symmetry of [k]. Ignore transverse shear deformation.

Problem #2 – 10pts

The plane structures shown consist of rigid weightless bars connected by linear springs with stiffness \( k \). Degrees of freedom are horizontal translation \( \theta_i \) for \( i=1,2 \) as shown. Vertical motion and out of plane displacements are not allowed. In each case, determine the structural stiffness matrix using principal of stationary potential energy.

Hint: Find stretch of each member in terms of DOFs and construct \( \Pi \).
Problem #3 – 12pts:

For the structure shown below, all joints are pinned except point (3) which is fully fixed. All members are bars except member 3-1 which is an Euler-Bernoulli beam. Cross section of all members are squares with side length of $t=5$cm and $E=1$GPa and Poisson ratio of 0.3. You don’t need to derive equations for the stiffness matrix. Just use the equations in the book.

A) Find the deformation of the bottom pin joint (1) under the shown forces using FEM approach.

B) Verify your results using Abaqus.

![Structure Diagram]

Problem #4 – 5pts:

A fin is a common example of a one-dimensional heat transfer problem. One end of the fin is connected to a heat source (whose temperature is known) and heat will be lost to the surroundings through the perimeter surface and the end. Energy balance states

$$k A_c \frac{dT}{dx} + h A_p (T-T_\infty) = 0$$

with $A_c, A_p$ being cross section and perimeter areas and $k$ and $h$ being conduction and convection coefficients. For a given fin of length $L$ and above parameters, Create shape functions and total heat transfer matrix $[K]_{\text{total}}$.

Problem #5 – 5pts: Exact solution of some equation has produced $z = \sin(x^2+y^2)$. Assume we want to approximate $z$ field for $x,y \in [.5,1]$ using a square 2D element with 1 DOF per node. Find $a_i$ coefficients for the fittings and plot both the fitted surface and original surface in the same 3D plot.