Homework #4

ME 441: Finite Elements
Department of Mechanical Engineering, University of Rochester
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Due Nov 12th in class

Problem #1 – 8pts:

6.3-7 Use Gauss quadrature to evaluate the integral

\[ I = \int_{-1}^{1} \int_{-1}^{1} \frac{3 + \xi^2}{2 + \eta^2} d\xi d\eta \]

Use (a) one, (b) four, and (c) nine Gaussian points. (d) Obtain exact solution and calculate percentage error of your numerical results compared against the exact solution.

Problem #2 – 5pts:

Derive locations and weights of an order 2 Gauss rule by requiring that it integrate exactly the polynomial \( \phi = a_1 + a_2 \xi + a_3 \xi^2 + a_4 \xi^3 \) in the interval \(-1 \leq \xi \leq 1\). Assume that sampling points and weights are symmetric with respect to the middle of the interval.

*Note: Assume we do not know those locations and weight factors from the lecture notes and the book. Basically, we want to prove order 2 Gauss rule can be exact for any order 3 polynomial.

Problem #3 – 16pts:

6.2-7 Evaluate [\( J \)] and \( J \) for each of the four elements shown. Also determine the ratio of element area to the area of a square two units on a side. How is this ratio related to \( J \), and why?

Problem #4 – 6 points

Show that the 7th and 8th mode shapes shown for the isoparametric Q4 elements indeed show zero strain at their center. Consider each side has an undeformed length of 1, and after
deformation, the sides shrink/stretch by 10%. You are expected to find nodal displacements based on this 10% deformation and find strain at any point (x,y), using shape functions.

Problem #5 – 8pts:

4.5-1 Verify that the first of Eqs. 4.5-6a is indeed given by the conditions $\partial \Pi_p / \partial a_1 = 0$ and $\partial \Pi_p / \partial a_2 = 0$.

4.5-2 Verify that the $a_i$ of Eq. 4.5-9 result from the use of the three-term polynomial $u = a_1 x + a_2 x^2 + a_3 x^3$ in a Rayleigh-Ritz solution.

Problem 6 – 10 points

Abaqus uses reduced integration as default for plane elements. To study the performance of these elements, consider a plane strain problem with modulus of elasticity of 1GPa and Poisson ratio of 0.3. The plane is subjected to a force in x direction equal to 1kN.

a) Using 10 divisions per side for seeding the mesh, obtain deformed shape of this structure under loading using the default element formulation and include a plot of your results. (Reduced integration)

b) Does this shape comply with the shape you expect, if you physically deform a plate under These loading and boundary conditions? What phenomenon does this represent? Using results from Problem 4, explain your answer

c) Fix this problem by manipulation of element formulation in Abaqus and include the deformed shape and total displacement contour in your report.